Constraint Satisfaction Problem
present state of the dichotomy conjecture

Paweł M. Idziak
Jagiellonian University, Krakow, Poland

A large class of problems in artificial intelligence, data basis and other areas of computer science can be viewed as Constraint Satisfaction Problems. For a given (finite) template (relational structure) $\mathbb{A} = (A; \mathcal{R})$ an instance of the Constraint Satisfaction Problem $CSP(\mathbb{A})$ is a primitive positive formula $\varphi(\overline{x})$ over the structure $\mathbb{A}$ and the question is whether this formula is satisfiable in $\mathbb{A}$.

Problem 1 (Constraint Satisfaction Problem)**.** Characterize relational structures $\mathbb{A}$ for which Constraint Satisfaction Problem $CSP(\mathbb{A})$ has poly-time solution.

There is a long standing conjecture of Feder and Vardi:

Conjecture 1 (Dichotomy Conjecture)**.** For every finite relational structure $\mathbb{A}$ the problem $CSP(\mathbb{A})$

- is either poly-time
- or NP-complete.

In last 10 years universal algebra started to play an important role in this area. This lecture will explain these connections and how universal algebra can help here. In fact, translating the relational structure $\mathbb{A} = (A; \mathcal{R})$ into a finite algebra $\mathbf{A} = (A; \mathcal{F})$, where $\mathcal{F}$ consists of all polymorphisms of $\mathcal{R}$, there is even stronger conjecture that actually describes where the borderline between structures of these two complexities lies:

Conjecture 2 (Strong Dichotomy Conjecture)**.** For every finite relational structure $\mathbb{A}$

- $CSP(\mathbb{A})$ is poly-time, if the corresponding algebra $\mathbb{A}$ has a nontrivial polymorphism, or equivalently $\mathbf{1} \notin \mathsf{typ}\,\{\mathsf{HSP}(\mathbf{A})\}$,
- $CSP(\mathbb{A})$ is NP-complete, otherwise.

The second item of this strong conjecture was easy to confirm. Now there is a desperate search for poly-time algorithms that make use of nontrivial polymorphisms. The present state of this fight will be presented.