The endomorphism semiring of a semilattice

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These results were obtained together with T. Kepka and M. Maróti. The question that we were interested in is: What are the semilattices M such that the semiring E_M of all endomorphisms of M (or some distinguished subsemirings of E_M) is simple, or at least subdirectly irreducible.

Denote by F_M the subsemiring of E_M generated by all at most two-valued endomorphisms of M. Then every subsemiring E of E_M containing F_M is subdirectly irreducible; if E contains the identical endomorphism, then E is simple if and only if M contains both the least and the largest elements. (We have $F_M = E_M$ if and only if M is a finite distributive lattice.)

Now let M be a semilattice with the largest element 1. We denote by E_M^1 the semiring of the endomorphisms f of M such that $f(M) = 1$; if M has also the least element 0, we denote by E_M^{01} the semiring of the endomorphisms f of M such that $f(0) = 0$ and $f(1) = 1$. It turns out that every subsemiring of E_M^1 containing all at most two-valued endomorphisms, and also every subsemiring of E_M^{01} containing all at most three-valued endomorphisms, is subdirectly irreducible. The description of their monoliths makes it possible to say precisely which of these subsemirings are simple.