

# An algebraic treatment of imprecise probabilities

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Joint work with Martina Fedel, Klaus Keimel and Walter Roth

Imprecise probabilities over fuzzy events are interpreted in terms of bets in de Finetti's style. That is, the upper probability of a fuzzy event  $\phi$  is the betting odd  $\alpha$  that a bookmaker would accept for the following bet: the bettor pays  $\alpha$  and gets the truth value of  $\phi$ . The lower probability of  $\phi$  is the amount  $\beta$  that a bookmaker would accept for the opposite bet (that is, the bookmaker pays  $\alpha$  and gets the truth value of  $\phi$ ).

Our rationality criterion is given for a whole book (a finite system of pairs event-betting odd) and not for a single bet: a book is *rational* if there is no bad bet, that is, there is no bet for which there is an alternative system of bets based on the book which gives the bettor a better payoff.

In collaboration with Martina Fedel, Klaus Keimel and Walter Roth we have proved that a book is rational iff it may be extended by an upper probability (i.e., a sublinear, monotonic, homogeneous and strongly normalized operator). The whole matter can be investigated in algebraic terms: events are represented as elements of a semisimple MV-algebra and upper and lower probabilities are represented as operators on the algebra.

In the last part of the talk, we plan to discuss the relationship with Walley's work on lower and upper prediction of gambles, as well as the possibility of an algebraizable logic for the treatment of all of these matters.