Free Modal Algebras: a Coalgebraic Perspective

Yde Venema
Universiteit van Amsterdam, Netherlands

A *modal algebra* is a Boolean algebra expanded with a *diamond*, that is, a unary operation $\Diamond$ which distributes over finite joins. These algebras are of interest because they provide the algebraic encoding of *modal logic*, a branch of logic with a wealth of applications, ranging from computer science to economics and linguistics. Among the many interesting properties of the *free* modal algebras, we will focus on the following:

(1) Similar to free distributive lattices carrying a Heyting implication, on free modal algebra one may explicitly define a *right adjoint* to $\Diamond$, that is, an unary operator $\blacksquare$ such that $\Diamond a \leq b \iff a \leq \blacksquare b$.

(2) The fact that basic modal logic enjoys *uniform* interpolation, is reflected by the existence of both left and right adjoints to the standard embedding of the κ-generated free modal algebra $\mathsf{M}(\kappa)$ into $\mathsf{M}(\kappa')$, for $\kappa < \kappa' < \omega$.

In this talk we will analyze these properties from a *coalgebraic* perspective, and generalize them to a wider coalgebraic setting.

First, we will reformulate the syntax and semantics of standard modal logic, based on the so-called cover modality ∇. We will introduce and underline the fundamental importance of an axiomatic law in this language which shows how conjunctions distribute over the cover modality. As an important application of this law, we will see how every modal formula can be rewritten into a distributive normal form, where the use of conjunctions is severely restricted. As some immediate consequences of this principle we will give a direct, inductive definition of the uniform interpolants mentioned above, and a direct definition of the operator $\blacksquare$.

Time permitting, we will discuss manifestations and repercussions of the mentioned axiomatic law in automata theory and in topology, and we finish the talk by putting the cover modality and its distributivity axiom into a wider coalgebraic perspective.