Finite distributive lattices with the antitone bijections on the principal filters

Michal Botur
e-mail: botur@inf.upol.cz
Palacký University in Olomouc, Czech Republic

The system $\mathbf{A} = (A, \vee, \wedge, (f_a)_{a \in A}, 1, 0)$ satisfying conditions

- $(A, \vee, \wedge, 0, 1)$ is a bounded lattice,
- f_a is antitone isomorphism on the interval $[a, 1]$ (for any $a \in A$), (more precisely, inequality $f_a(x) \leq f_a(y)$ holds if and only if $y \leq x$ holds for any $x, y \in A$).

is called to be a lattice with section antitone bijections.

Moreover, if the mappings f_a are involutions (thus $f_a(f_a(x)) = x$ holds for any $a \in A$ and $x \in [a, 1]$) then the system is called to be a lattice with section antitone involutions.

The basic algebras was introduced in [2] as algebraic representation of the lattices with section antitone involutions. We recall that the algebra $\mathbf{A} = (A, \oplus, \neg, 0)$ of type $\langle 2, 1, 0 \rangle$ is a basic algebra if it satisfies the identities

BA1 $x \oplus 0 = x$,
BA2 $\neg\neg x = x$,
BA3 $\neg(\neg x \oplus y) \oplus y = \neg(\neg y \oplus x) \oplus x$,
BA4 $\neg(\neg(\neg(x \oplus y) \oplus y) \oplus z) \oplus (x \oplus z) = 1$.

It was proved in [1] that a finite comutative basic algebras (the basic algebras with a commutative operation '$\oplus$') are just a finite MV algebras (the operation '$\oplus$' is also associative), see [3].

We will prove the theorem which is motivated by previous results:

Theorem 1. *If the system* $\mathbf{A} = (A, \vee, \wedge, (f_a)_{a \in A}, 1, 0)$ *is a lattice with section antitone bijections such that* $(A, \vee, \wedge)$ *is distributive lattice then the lattice* $(A, \vee, \wedge)$ *is a direct product of the finite chains.*

References

[1] M. Botur, R. Halaš: Finite commutative basic algebras are MV-algebras, Mult. Val. Logic and Soft Comp. **14(1-2)** (2008), 69-80.
[2] I.Chajda, R. Halaš, J.Kühr: *Multiple Valued Quantum Algebras*, Algebra Universalis, to appear.
[3] R. L. O. Cignoli, M.L. D'Ottaviano, D. Mundici: *Algebraic Foundations of Many-valued Reasoning*, Kluwer Acad. Publ., Dordrecht, 2000.