Countable Random p-Groups with Prescribed Ulm-Invariants

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We present a probabilistic construction of countable abelian p-groups with prescribed Ulm-sequence. Our main result provides a different proof for the existence theorem of abelian p-groups with any given countable Ulm-sequence due to Ulm (1933), which is sometimes called Zippin's theorem. The basic idea, applying probabilistic arguments, comes from a result by Erdős and Rényi (1963). They gave an amazing probabilistic construction of countable graphs which with probability 1, produces the universal homogeneous graph, therefore also called the random graph. Cameron says about this in his book "Oligomorphic Permutation Groups" (1990, pp. 86,87): *In 1963, Erdős and Rényi proved the following paradoxical result. ... It is my contention that mathematics is unique among academic pursuits in that such an apparently outrageous claim can be made completely convincing by a short argument.* The algebraic tool in the present paper needs methods developed in the 70th of the last century, the theory of valuated abelian p-groups. Valuated abelian p-groups are natural generalizations of abelian p-groups with the height valuation, investigated in detail by Richman and Walker (1973, 1979) and others. We have to establish extensions of finite valuated abelian p-groups dominated by a given Ulm-sequence f. Our probabilistic construction produces a countable valuated abelian p-group with Ulm sequence dominated by f. Our main result shows that with probability 1, the constructed group has precisely f as its Ulm sequence.

Probabilistic results of similar nature for Scott-domains and for causal sets were presented (as invited lectures) at AAA77 (2009, by the author) and a topological version at AAA73 (2008) by M. Goldstern.