Absolute E-Rings

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A ring R with 1 is called an *E-ring* if $\mathrm{End}_{\mathbb{Z}}R$ is ring-isomorphic to R under the canonical homomorphism taking the value 1σ for any $\sigma \in \mathrm{End}_{\mathbb{Z}}R$. Moreover R is an *absolute E-ring* if it remains an E-ring in every generic extension of the universe. E-rings are an important tool for algebraic topology as explained in my talk. The existence of an E-ring R of each cardinality of the form $\lambda^{\aleph_0}$ was shown by Dugas, Mader and Vinsonhaler [1]. We want to show the existence of absolute E-rings. It turns out that there is a precise cardinal-barrier $\kappa(\omega)$ - the first ω-Erdős cardinal - for this problem. (This is a relative of measurable cardinals.) We will construct absolute E-rings of size $\lambda < \kappa(\omega)$. But there are no absolute E-ring of cardinality $\geq \kappa(\omega)$. The non-existence of huge, absolute E-rings $\geq \kappa(\omega)$ follows from a recent paper by Herden and Shelah [5] and the construction of absolute E-rings R is based on an old result by Shelah [6] where families of absolute, rigid colored trees (with no automorphism between any distinct members) are constructed. We plant these trees into the potential E-rings with the aim to prevent unwanted endomorphisms of their additive group to survive. Endomorphisms will recognize the trees which will have branches infinitely often divisible by primes. Our main results provides the existence of E-rings for all infinite cardinals λ, and if $\lambda < \kappa(\omega)$, then the construction is absolute, i.e. these E-rings remain E-rings in all generic extensions of the universe (e.g. using forcing arguments). This result will appear in [3]. Indeed *all* previously known E-rings ([1],[4]) of cardinality $\geq 2^{\aleph_0}$ have a free additive group R^+ in some extended universe, thus are no longer E-rings, as follows by the Levy collapse. Our construction also fills all cardinal-gaps of the earlier constructions (which have only sizes $\lambda^{\aleph_0}$). These E-rings are domains and as a by-product we obtain the existence absolutely of indecomposable abelian groups, compare [4].

References

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