Constraint Satisfaction Problem

present state of the dichotomy conjecture, continued

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The talk will be a a continuation of Pawel Idziak's lecture. I will provide more in–depth information on the algebraic tools and techniques developed in order to tackle the Dichotomy Conjecture of Feder and Vardi.

During the first part of the lecture I will present algorithms solving CSPs in restricted settings:

- the local consistency checking algorithm together with an algebraic characterization of its applicability;
- the "Dalmau's algorithm" together with an algebraic characterization of its applicability;

and during the second part of the lecture I will present algebraic tools which are being developed with a unification of these algorithms in mind:

- the characterization of finitely generated Taylor varieties via cyclic terms;
- the notion of "absorbing subuniverses" and it's applicability.

The notions of the local consistency checking algorithm and the Dalmau's algorithm will be introduced during the lecture of Pawel Idziak. The first new notion of the lecture is the notion of a cyclic term

Definition 1. A term $t(x_1, \ldots, x_n)$ is a *cyclic term* of an algebra $\mathbf{A}$ if it is idempotent and

$$t(x_1, x_2, \ldots, x_n) \approx t(x_2, \ldots, x_n, x_1)$$

holds in $\mathbf{A}$.

which characterize finite algebras generating varieties with Taylor terms

Theorem 2. *A finite algebra generates a variety with a Taylor term, if and only if, it has a cyclic term.*

A second new notion, which also provides a characterization of Taylor varieties, is a notion of an *absorbing subuniverse*:

Definition 3. A subuniverse $\mathbf{B}$ of an idempotent algebra $\mathbf{A}$ is an *absorbing subuniverse* if and only if there exists a term $t(x_1, \ldots, x_n)$ such that for any choice of arguments $a_1, \ldots, a_n \in A$

$$\text{if, for all but one } i \text{, we have } a_i \in B \text{ then } t(a_1, \ldots, a_n) \in B.$$

The characterization of finitely generated Taylor varieties in terms of absorbing subuniverses will be presented during the talk.