Homomorphism-homogeneous lattices and semilattices

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In 2006 P. J. Cameron and J. Nešetřil introduced a relaxed version of homogeneity: we say that a structure is homomorphism-homogeneous if every homomorphism between finitely generated substructures of the structure extends to an endomorphism of the structure.

In this talk we consider homomorphism-homogeneous lattices and homomorphism-homogeneous semilattices understood as algebras. We present the characterization of homomorphism-homogeneous lattices, and then provide several classes of homomorphism-homogeneous semilattices and several classes of semilattices which are not homomorphism-homogeneous. Our main results are:

Theorem 1. *A lattice L is homomorphism-homogeneous if and only if it is either a chain, or every interval of L is a Boolean lattice.*

Theorem 2. *A finite lattice L is homomorphism-homogeneous if and only if it is either a chain, or a direct power of the 2-element chain.*

Theorem 3. *Every tree is a homomorphism-homogeneous semilattice.*

Let T be a tree, and let T^* be a semilattice obtained from T by adjoining to it a top element. Then T^* is homomorphism-homogeneous.

Theorem 4. *Let $(L, \wedge, \vee)$ be a distributive lattice. Then $L_\wedge$, the $\wedge$-reduct of L, is a homomorphism-homogeneous semilattice.*

The characterization of homomorphism-homogeneous semilattices is still an open problem.