Complexity of quasivariety lattices of pointed Abelian groups

Anvar Nurakunov
e-mail: a.nurakunov@gmx.net
Institute of Theoretical and Applied Mathematics, Bishkek, Kyrgyzstan

A *quasivariety* is a universal Horn class of algebraic structures containing the trivial structure. The set $Lq(\mathcal{R})$ of all subquasivarieties of a quasivariety $\mathcal{R}$ forms a complete lattice under inclusion. A lattice isomorphic to $Lq(\mathcal{R})$ for some quasivariety $\mathcal{R}$ is called a *lattice of quasivarieties* or *quasivariety lattice*. The Birkhoff-Maltsev Problem asks which lattices are isomorphic to lattices of quasivarieties.

A quasivariety $\mathcal{R}$ is $Q-universal$ providing that, for every quasivariety $\mathcal{K}$, the lattice $Lq(\mathcal{K})$ is a homomorphic image of some sublattice of $Lq(\mathcal{R})$. A lattice L is *obstructive* if the set of all finite sublattices of L is not computable, that is, there is no algorithm for deciding whether a finite lattice is a sublattice of L. An algebra A is called pointed Abelian group if its signature consists binary operation $+$, unary operation $-$ and two constants $0, c$ such that $< +, -, 0 >$-reduct is Abelian group. The main results state that quasivariety of pointed Abelian groups is Q-universal and there are uncountable obstructive quasivariety lattices of pointed Abelian groups.