Hyper-Pseudoformulas and M-Solid Ordered Pseudovarieties

Klaus Denecke
e-mail: kdenecke@rz.uni-potsdam.de
Potsdam University, Germany

Dara Phusanga*
e-mail: p.phu2514@hotmail.com
Potsdam University

In [2] was proved that for a monoid $\mathcal{M}$ of hypersubstitutions M-solid positive varieties of tree languages correspond to M-solid ordered pseudovarieties. Therefore, it is necessary to have a characterization of M-solid peudovarieties In this paper we will characterize M-solid ordered pseudovarieties in a similar way in which in [5] M-solid varieties, in [1] M-solid quasivarieties, in [3] M-solid pseudovarieties and in [4] M-solid algebraic systems were characterized. The main idea is to show that we have two Galois-connections and a conjugate pair of additive closure operators. Then we can apply the general theory of conjugate pairs of additive closure operators to obtain a characterization.

It will be presented how we apply the theory of conjugate pairs of additive closure operators(see [6]) to finite ordered algebras and pseudoformulas. The presentation will be divided into four parts. In the first part we will introduce some basic concepts on finite ordered algebras. We describe classes of finite ordered algebras as model classes of logical sentences, we need the concept of implicit operations on an ordered pseudovariety. In the second part we will define pseudoformulas of type $(\tau,(2))$ by implicit operations(see [7]) on an ordered pseudovariety $V_{\leq}$ and we define the satisfaction of a pseudoformula of type $(\tau,(2))$ on $V_{\leq}$ by the ordered algebra $\mathcal{A}^{\leq}$, written as $\mathcal{A}^{\leq} \underset{p.s.}{\models} PF$. From this relation $\underset{p.s.}{\models}$ we get the first Galois-connection (PSM, PSF) between classes of finite ordered algebras of the same type and collections of pseudoformulas. Then on the third part, we will use the concept of a hypersubstitution $\sigma_H \in Hyp(\tau)$ as in [3]. A mapping $\bar{\sigma}_H : \overline{\Omega}_n V_{\leq} \to \overline{\Omega}_n V_{\leq}$ can be defined by $\bar{\sigma}_H(\pi) := \lim\limits_{k\to\infty}(\hat{\sigma}_H[t_k]^{\mathcal{A}})_{\mathcal{A}\in V}$. This mapping will be used to define hypersubstitutions for pseudoformulas of type $(\tau,(2))$. Using hypersubstitutions for pseudoformulas of type $(\tau,(2))$ we define hyper-pseudoformulas. Satisfaction of a hyper-pseudoformula by an ordered algebra ,written as $\mathcal{A}^{\leq} \underset{h.p.s.}{\models} PF$ defines a second Galois-connection $(HPSM, HPSF)$ between classes of finite ordered algebras of the same type and collections of pseudoformulas. A M-solid ordered pseudovariety is a pseudovariety of finite ordered algebras which is closed under taking of so-called derived finite ordered algebras.

Finally - on part 4 - all conditions to apply the general theory of conjugate pairs of additive closure operators are satisfied and we can characterize M-solid ordered pseudovarieties. To do this we apply the characterization theorem from [6].

References

[1] Chompoonut, Ch. and Denecke, K., : *M-solid Quasivarieties*, East-West Journal of Math., Vol. 4, No. 2, (2002) 177-190.

[2] Denecke, K. and Koppitz, J., : *M-solid Positive Varieties of Tree Languages*, preprint 2009.

[3] Denecke, K. and Pibaljommee, B., : *Pseudoidentities and Hyper-pseudoidentities*, in: Advances in Algebra towards Millenium Problems, ed. by Ki-Bong Nam et. al., SAS International Publications, 2005, pp 163-180.

[4] Denecke, K. and Phusanga, D., : *Hyperformulas and Solid Algebraic Systems*, Studia Logica, Vol. 90, No. 2, (2008), 263-286.

[5] Denecke, K. and Reichel, M., : *Monoids of Hypersubstitutions and M-solid Varieties*, Contributions to General Algebra 9, Verlag Hlder-Pichler-Temsky, Wien 1995, Verlag B. G. Teubner, Stuttgart, pp. 117-126.

[6] Koppitz, J. and Denecke, K., : *M-solid Varieties*, Springer, 2006.

[7] Reiterman, J., : *The Birkhoff theorem for finite algebras*, Algebra Universalis, 14 (1982) 1-10.