On Commutators and Associators in Catalan loops

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Catalan loops as defined by Long and Smith are two-sided loops which originated from an issue in number theory, studying the relationship between Fermat curves and modular curves. (A more detailed description of the motivation is given in [1].)

Various commutators and associators may be defined in one-sided loops. In this talk, we will first give a general discussion of commutators and associators in left and right loops. Then we will approximate and compare these objects in the left and right lo op reducts of a Catalan loop. To within a certain order of approximation, they turn out to be quite symmetrical. Using the general analysis of commutators and associators, we investigate the structure of a specific Catalan loop which is non-commutative, bu t associative, that appears in the original number-theoretic application of Catalan loops. There is still a lot of work to be done to apply this to the general case, but this talk hopes to provide a good start.

References

[1] Long L., Smith J.D.H. : Catalan Loops, preprint, 2009.