TABLICE – TRANSFORMACJA FOURIERA

1. Własności TF funkcji – niech $f\colon \mathbb{R}^n \to \mathbb{C}$, $\widehat{f} = \mathcal{F}[f]$, spełnia odpowiednie założenia

funkcja $g(\mathbf{x})$	transformata $\widehat{g}(\boldsymbol{\xi}) = \mathcal{F}[g](\boldsymbol{\xi})$
$f(\mathbf{A}\mathbf{x}), \qquad \mathbf{A} \in \mathbb{R}^{n\times n}$	$\dfrac{1}{\lvert\det \mathbf{A}\rvert}\widehat{f}(\mathbf{A}^{-T}\boldsymbol{\xi})$
$f(\mathbf{x}-\mathbf{x}_0), \qquad \mathbf{x}_0 \in \mathbb{R}^n$	$\mathrm{e}^{-2\pi i \mathbf{x}_0\cdot\boldsymbol{\xi}}\widehat{f}(\boldsymbol{\xi})$
$\mathrm{e}^{2\pi i \boldsymbol{\xi}_0\cdot\mathbf{x}} f(\mathbf{x}), \qquad \boldsymbol{\xi}_0 \in \mathbb{R}^n$	$\widehat{f}(\boldsymbol{\xi}-\boldsymbol{\xi}_0)$
$\overline{f(\mathbf{x})}$	$\overline{\widehat{f}(-\boldsymbol{\xi})}$
$f_{x_k}(\mathbf{x})$	$2\pi i \xi_k \cdot \widehat{f}(\boldsymbol{\xi})$
$-2\pi i x_k \cdot f(\mathbf{x}),$	$\widehat{f}_{\xi_k}(\boldsymbol{\xi})$
$(f_1 * f_2)(\mathbf{x})$	$\widehat{f_1}(\boldsymbol{\xi}) \cdot \widehat{f_2}(\boldsymbol{\xi})$

$\displaystyle\int_{\mathbb{R}^n} f(\mathbf{x})\cdot\widehat{g}(\mathbf{x})\,\mathrm{d}\mathbf{x} = \int_{\mathbb{R}^n}\widehat{f}(\mathbf{x})\cdot g(\mathbf{x})\,\mathrm{d}\mathbf{x}$	$\displaystyle\int_{\mathbb{R}^n} \lvert f(\mathbf{x})\rvert^2\,\mathrm{d}\mathbf{x} = \int_{\mathbb{R}^n}\lvert\widehat{f}(\mathbf{x})\rvert^2\,\mathrm{d}\mathbf{x}$

2. Własności TF dystrybucji temperowanych – niech $f \in \mathcal{S}'(\mathbb{R}^n)$, $\widehat{f} = \mathcal{F}[f]$

Oznaczenia:

$$\forall_{f\in\mathcal{D}'(\mathbb{R}^n)}\,\forall_{\varphi\in\mathcal{D}(\mathbb{R}^n)} \quad \langle \beta f, \varphi\rangle = \langle f, \beta\varphi\rangle, \qquad \text{gdzie } \beta \in \mathcal{D}(\mathbb{R}^n);$$

$$\forall_{f\in\mathcal{D}'(\mathbb{R}^n)}\,\forall_{\varphi\in\mathcal{D}(\mathbb{R}^n)} \quad \langle T_{\mathbf{x}_0} f, \varphi\rangle = \langle f, T_{-\mathbf{x}_0}\varphi\rangle, \qquad \text{gdzie } \mathbf{x}_0 \in \mathbb{R}^n,\ T_{\mathbf{x}_0}\varphi(\mathbf{x}) = \varphi(\mathbf{x}-\mathbf{x}_0);$$

$$\forall_{f\in\mathcal{D}'(\mathbb{R}^n)}\,\forall_{\varphi\in\mathcal{D}(\mathbb{R}^n)} \quad \langle \check{f}, \varphi\rangle = \langle f, \check{\varphi}\rangle, \qquad \text{gdzie } \check{\varphi}(\mathbf{x}) = \varphi(-\mathbf{x});$$

$(\mathrm{D}^\alpha f)\widehat{\ } = (2\pi i\boldsymbol{\xi})^\alpha\widehat{f}$	$((-2\pi i\mathbf{x})^\alpha f)\widehat{\ } = \mathrm{D}^\alpha\widehat{f}$
$(T_{\mathbf{x}_0} f)\widehat{\ } = \mathrm{e}^{-2\pi i\mathbf{x}_0\cdot\boldsymbol{\xi}}\widehat{f}$	$(\mathrm{e}^{2\pi i\mathbf{x}\cdot\boldsymbol{\xi}_0} f)\widehat{\ } = T_{\boldsymbol{\xi}_0}\widehat{f}$
$(\check{f})\widehat{\ } = (\widehat{f})\check{\ }$	$(f)\widehat{\widehat{\ }} = \check{f}$

3. Pary transformat – niech $\alpha \in \mathbb{R}_+$, $\xi_0 \in \mathbb{R}$.

funkcja $f(x)$	transformata $\widehat{f}(\xi) = \mathcal{F}[f](\xi)$	dystrybucja $f \in \mathcal{S}'(\mathbb{R})$	transformata $\widehat{f} = \mathcal{F}[f]$
$\mathrm{e}^{-\pi x^2}$	$\mathrm{e}^{-\pi \xi^2}$	δ	1
$\mathbb{1}_{(-\alpha,+\alpha)}(x)$	$2\alpha \dfrac{\sin(2\pi\alpha\xi)}{2\pi\alpha\xi}$	1	δ
$\dfrac{\sin(2\pi\alpha x)}{2\pi\alpha x}$	$\dfrac{1}{2\alpha}\mathbb{1}_{(-\alpha,+\alpha)}(\xi)$	δ_{x_0}	$\mathrm{e}^{-2\pi i x_0 \xi}$
$\Lambda_{(-\alpha,+\alpha)}(x)$	$\alpha \left(\dfrac{\sin(\pi\alpha\xi)}{\pi\alpha\xi}\right)^2$	$\mathrm{e}^{2\pi i x \xi_0}$	δ_{ξ_0}
$\left(\dfrac{\sin(2\pi\alpha x)}{2\pi\alpha x}\right)^2$	$\dfrac{1}{2\alpha}\Lambda_{(-2\alpha,+2\alpha)}(\xi)$	$P.V.\dfrac{1}{x}$	$-i\pi \operatorname{sgn} \xi$
$\mathrm{e}^{-\alpha\|x\|}$	$\dfrac{2\alpha}{\alpha^2 + (2\pi\xi)^2}$	$\cos(2\pi x \xi_0)$	$\dfrac{1}{2}(\delta_{\xi_0} + \delta_{-\xi_0})$
$\dfrac{1}{\alpha^2 + x^2}$	$\dfrac{\pi}{\alpha}\mathrm{e}^{-2\pi\alpha\|\xi\|}$	$\sin(2\pi x \xi_0)$	$\dfrac{1}{2i}(\delta_{\xi_0} - \delta_{-\xi_0})$

* gdzie $\mathbb{1}_{(-\alpha,+\alpha)}(x) = \begin{cases} 1 & \text{dla } |x| < \alpha, \\ \frac{1}{2} & \text{dla } |x| = \alpha, \\ 0 & \text{w p.p.} \end{cases}$ oraz $\Lambda_{(-\alpha,+\alpha)}(x) = \begin{cases} x + \alpha & \text{dla } \alpha \leqslant x \leqslant 0, \\ -x + \alpha & \text{dla } 0 < x \leqslant \alpha,. \\ 0 & \text{w p.p.} \end{cases}$