

1. Oblicz granicę ciągu liczbowego:

- (a) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{\sqrt{n}}\right)^n$,
- (b) $\lim_{n \rightarrow \infty} \left(\frac{2n^2 + 2n + 1}{2n^2 + 2}\right)^{n+1}$,
- (c) $\lim_{n \rightarrow \infty} \left(\frac{n+1}{n}\right)^{2019}$,
- (d) $\lim_{n \rightarrow \infty} n \cdot [\ln(n+3) - \ln n]$,
- (e) $\lim_{n \rightarrow \infty} \left(\frac{n^2 + 2n + 1}{2n^2 + n + 1}\right)^{\frac{n+1}{n-1}}$,
- (f) $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{4n+1}\right)^{5n-2}$.

2. Wykaż, że nie istnieje granica:

- (a) $\lim_{n \rightarrow \infty} \frac{\cos(n\pi) + \sqrt{3}}{2 \cos(n\pi) + \sqrt{2}}$,
- (b) $\lim_{n \rightarrow \infty} \frac{n-1}{n \left[\sin\left(\frac{n\pi}{2}\right) + 2 \cos\left(\frac{n\pi}{2}\right) + 3\right]}$,
- (c) $\lim_{n \rightarrow \infty} a^n$, gdzie $a \in \mathbb{R}$ i $a < -1$.

3. Oblicz granicę funkcji:

- (a) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x}{x}$,
- (b) $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x^2}$,
- (c) $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{3x}$,
- (d) $\lim_{x \rightarrow 0} (1 - \sin x)^{\frac{1}{x}}$,
- (e) $\lim_{x \rightarrow 0} (\cos x)^{\operatorname{ctg}^2 x}$,
- (f) $\lim_{x \rightarrow 0} \frac{\sin 4x}{4 - \sqrt{5x + 16}}$,
- (g) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\cos 3x}$,
- (h) $\lim_{x \rightarrow -1} \frac{\sin(x^2 - 1)}{x + 1}$,
- (i) $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + x + 1}}{x}$,
- (j) $\lim_{x \rightarrow -\infty} \frac{\sqrt{2-x} - \sqrt{1-x}}{x + \sqrt{x^2 + 2x + 3}}$,

$$(k) \lim_{x \rightarrow \infty} x \cdot \operatorname{arctg} x.$$

4. Wykaż, że nie istnieje granica:

$$(a) \lim_{x \rightarrow -\infty} \sin 3x,$$

$$(b) \lim_{x \rightarrow 0^+} \cos\left(\frac{2}{x}\right),$$

$$(c) \lim_{x \rightarrow 0} (1 + |x|)^{\frac{1}{x}},$$

$$(d) \lim_{x \rightarrow 1} 2^{\frac{1}{x-1}},$$

$$(e) \lim_{x \rightarrow 0} \sin\left(\operatorname{arctg} \frac{1}{x}\right).$$