On the cyclic coloring conjecture

Stanislav Jendroľ

(joint work with Roman Soták)

A cyclic coloring of a plane graph G is a coloring of its vertices such that vertices incident with the same face have distinct colors. The minimum number of colors in a cyclic coloring of a plane graph G is its cyclic chromatic number $\chi_c(G)$. For a 2-connected plane graph G let $R(G)$ be the graph (called the reduction of G) obtained from G by replacing all maximal paths all interior vertices of which have degree 2 with edges.

We show that the Cyclic Coloring Conjecture of Borodin from 1984, saying that every connected plane graph G has $\chi_c(G) \leq \lfloor \frac{3}{2}\Delta^*(G) \rfloor$, can be reduced to hold for 2-connected plane graphs G whose reductions $R(G)$ are simple 3-connected plane graphs. We have received four different upper bounds for graphs G from this restricted family. Moreover, we have proved that the conjecture of Borodin holds for 2-connected plane graphs with a large maximum face degree and for two wide families of plane graphs.