

Directed Cages and the Caccetta–Haggkvist Conjecture

Adrian Bondy

Université Pierre et Marie Curie, Paris 6

The Caccetta–Haggkvist Conjecture asserts that every oriented graph on n vertices in which each vertex has outdegree at least d contains a directed cycle of length at most $\lceil n/d \rceil$. This conjecture generalizes one on directed cages proposed over thirty years ago by M. Behzad, G. Chartrand and C.E. Wall. The conjecture has been confirmed for small values of d . At the other extreme, when $d \geq n/3$, the problem of concluding that there exists a directed triangle has proved to be remarkably elusive. This special case has given rise to several related questions, notably Seymour’s intriguing Second Neighbourhood Conjecture. We shall present an overview of the current state of knowledge on the topic, focusing on proof techniques, examples and counterexamples.