Plane curves

9.1 Find tangent and normal line to the curve $(x^2+y^2)x - ay^2 = 0, \quad a \neq 0,$ at point $P\left(\frac{a}{2}, \frac{a}{2}\right)$.

9.2 Line $y = 3x - 5$ is tangent to the parabola $y = x^2 + bx + c$ at point $x = 2$. Find b and c.

9.3 Find the equation of tangent line to the curve $x = t^2 - 1, \quad y = t^3 + 1$ parallel to the line $2x - y + 3 = 0$

9.4 Show that the angle between tangent line to the curve $y = x^5 + 2x^3 + x + 1$ and x axis is in the interval $\left\langle \frac{\pi}{4}; \frac{\pi}{2} \right)$

9.5 Show that the distance between the normal line to $x = a(\cos t + t \sin t), \quad y = a(\sin t - t \cos t)$ and origin is equal for every normal line.

9.6 Find the degree of tangency at the origin:

- $y_1 = x^3, \quad y_2 = x \sin x$
- $y_1 = \sin x, \quad y_2 = x^4 - \frac{1}{6}x^3 + x$

9.7 Find point P and parameters a and b such that curve $y = ax + b$ and $y = x^3 - 3x^2 + 2$ are tangent at P and the degree is equal to 2.

9.8 Find curve tangent to the $y = \sin x$ at $P(0,0)$ from family of curves $y = ax^3 + bx^2 + cx + d$. with the highest degree of tangency. What is the degeree of tangency?

9.9 Find a, b and c such that the parabola $y = ax^2 + bx + c$ was tangent to the curve $y = x^3$ at point $P(1,1)$ with highest degeree.

9.10 Find the radius and center of curvature of the curve $y = x^4 + x^2 - \frac{1}{2}$ at point $P(0, -\frac{1}{2})$.

9.11 Fint the highest curvature of the curve $y = \ln x$