

Complex numbers

1.1 Calculate:

a) $(1 - 3i) + (4 - i)$

b) $(1 + 2i) - (3 - 6i)$

c) $(1 - i)(6 + 5i)$

d) $\frac{2 + 3i}{1 - i}$

e) $(1 + 3i)(1 - 3i)$

f) $(2 + i)^2$

g) $(\sqrt{3} - i)^3$

h) $\frac{1}{2 - i}$

i) $\frac{1}{(1 - 2i)^2}$

1.2 Find real numbers x and y that satisfy equations:

a) $x(2 + 3i) + y(5 - 2i) = -8 + 7i$

b) $(2 + yi) \cdot (x - 3i) = 7 - i$

c) $\frac{1 + yi}{x - 2i} = 3i - 1$

d) $\frac{x + yi}{x - yi} = \frac{9 - 2i}{9 + 2i}$

1.3 Solve equations:

a) $z^2 = 4\bar{z}$

b) $\frac{1 + i}{z} = \frac{2 - 3i}{\bar{z}}$

c) $2z + \bar{z} = 6 - 5i$

d) $(z + 2)^2 = (\bar{z} + 2)^2$

1.4 Calculate modulus of complex numbers:

a) $4i$

b) $12i - 5$

c) $(4i + 3)(\sqrt{2} - i)$

d) $\frac{2 - i}{\sqrt{3}i - 1}$

e) $\overline{\sqrt{5} + 2i}$

f) $(1 - \sqrt{2}i)^4$

1.5 Draw sets of numbers on complex plane:

a) $\operatorname{Re}(iz + 2) \geq 0$

b) $\operatorname{Im} z^2 < 0$

c) $\overline{z - i} = z - 1$

d) $\frac{4}{z} = \bar{z}$

e) $|z - 3 + 4i| = 1$

f) $\left| \frac{z - 2i}{z + 1} \right| = 1$

g) $2 \leq |iz - 5| < 3$

h) $|z + 1 - 2i| \geq 3$ and $|z - 3| < 4$

i) $\left| \frac{z + i}{z^2 + 1} \right| \geq 1$