

Complex numbers and polynomials

3.1 Using exponential form solve equations:

a) $z^7 = \bar{z}$

b) $\bar{z}^4 = z^2|z^2|$

c) $(\bar{z})^2|z^2| = \frac{4}{z^2}$

d) $|z|^3 = iz^3$

e) $z^6 = (\bar{z})^6$

f) $|z^8| = z^4$

3.2 Divide P over Q and write the quotient and remainder

a) $P(z) = 2z^4 - 3z^3 + 4z^2 - 5z + 6, \quad Q(z) = z^2 - 3z + 1$

b) $P(z) = z^{16} - 16, \quad Q(z) = z^4 + 2$

3.3 Solve equations

a) $z^2 - 4z + 13 = 0$

b) $z^2 - (3 - 2i)z + (5 - 5i) = 0$

c) $z^4 + 8z^2 + 15 = 0$

d) $(1 + i)z^2(6 + 2i)z + 142i = 0$

3.4 Find all roots of polynomial if you know one of them:

a) $P(z) = z^3 - 3\sqrt{2}z^2 + 7z - 3\sqrt{2}, \quad z_1 = \sqrt{2} + i$

b) $P(z) = z^4 - 6z^3 + 7z^2 + 6z - 30, \quad z_1 = 1 - 3i$

c) $P(z) = z^4 - 6z^3 + 18z^2 - 30z + 25, \quad z_1 = 2 + i$