Differential equations with separable variables

1.1 Verify that y is a solution of the ODE. Determine from y the particular solution of the initial value problem.

a) $y' + 5xy = 0,\ y = Ce^{-\frac{5}{2}x^2},\ y(0) = \pi$

b) $y' = y + e^x,\ y = (x + C)e^x,\ y(0) = \frac{1}{2}$

c) $yy' = 4x,\ y^2 - 4x^2 = C\ (y > 0),\ y(1) = 4$

d) $y' \operatorname{tg} x = 2y - 8,\ y = C \sin^2 x + 4,\ y(\frac{\pi}{2}) = 0$

1.2 Find the solution of differential equations with separable variables:

a) $\dfrac{dy}{dx} = e^{2x+y} \qquad y(1) = \ln(2)$

b) $xy' + y = y^2$

c) $ydy + xdx = 3xy^2dx \qquad y(2) = 1$

d) $2x\sqrt{1 - y^2}dx + ydy = 0$

e) $\sin x \cos 2y\ dx + \cos x \sin 2y\ dy = 0 \qquad y(0) = \frac{\pi}{2}$

f) $y' = xy^2 + x$

g) $(1 - x)dy = x(y + 1)dx \qquad y(0) = 0$

h) $x^2(y + 1)^3dx = (1 + x)^3y^3dy$

i) $(y^2 + xy^2)dx + (x^2 - x^2y)dy = 0$

1.3 Find the solution of differential equations with homogeneous coefficients:

a) $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{xy}$

b) $\dfrac{dy}{dx} = \dfrac{x + y}{3x - y}$

c) $y^2 + x^2y' = xyy' \qquad y(1) = 1$

d) $(y^2 - 3x^2)\, dy + 2xydx = 0$

e) $y - xy' = x + yy'$

f) $x^2 + 2xy - y^2 + (y^2 + 2xy - x^2)\, y' = 0$