

Problems leading to differential equations

3.1 Find the equation of the family of curves that satisfy the property described:

- a) The area of the region bounded by the x axis, the tangent line drawn at the point $P(x, y)$ of the curve of the family and the projection of the tangent line on the x-axis has a constant value A .
- b) the slope of the tangent at each point of a curve is equal to the difference between y and x coordinate. Find the particular curve through the origin.
- c) The segment of a normal line between the curve and the y axis is bisected by x axis. Find the particular curve through the point $(4, 2)$.
- d) The normal and the line drawn to the origin from the point of contact of the normal and the curve form an isosceles triangle with the x axis.
- e) The tangent and the line drawn to the origin from the point of tangency form an isosceles triangle with the x axis.

3.2 Find the orthogonal trajectories of each of the following family of curves

- a) $x^2 + y^2 - 2Cx = 0$
- b) $y^2 = 2(x - C)$
- c) $(y - 2)^2 = 2Cx$
- d) $y = Cx^2$
- e) $x^2 + 2y^2 = C$
- f) $xy = C$