Linear differential equations

5.1 Solve equations:

a) $y'' + 3y' + 2y = 4$

b) $y'' - y' = 3x^2$

c) $y'' + 2y' + y = x^2 e^{-x}$

d) $y'' - 6y' + 9y = 3x - 8e^x$

e) $y^{(4)} - 2y''' + y'' = x + xe^x$

f) $y'' - 2y' + 10y = 37\cos 3x$

g) $y'' + y = \sin^2 x$

h) $y'' - 3y' + 2y = \dfrac{e^{3x}}{1 + e^{2x}}$

i) $y'' - 2y' + y = \dfrac{e^x}{x^2}$

j) $y'' + y = \operatorname{tg} x + xe^x$

k) $y'' + 4y = \dfrac{1}{\sin^2 x} + 4x^2 + 4e^{2x} + 4\sin 2x$

5.2 $w(\lambda)$ is a characteristic polynomial of inhomogeneous linear differential equation. Write the trial particular solution if $Q(x)$ is equal to:

I. $w(\lambda) = \lambda^2(\lambda - 1)(\lambda^2 + 4)(\lambda^2 - 2\lambda + 2)$

a) $Q(x) = x^2 + 1$ **b)** $Q(x) = xe^x$ **c)** $Q(x) = \sin x$

d) $Q(x) = \cos 2x$ **e)** $Q(x) = e^x \cos x$ **f)** $Q(x) = 2$

g) $Q(x) = x \sin x$ **h)** $Q(x) = e^x \cos 2x$ **i)** $Q(x) = x^2 e^{3x}$

II. $w(\lambda) = \lambda^3(\lambda - 1)^2(\lambda - 2)(\lambda^2 + 1)^2$

a) $Q(x) = x^2 + \cos x + 3$ **b)** $Q(x) = x^2 e^x + x + 1$ **c)** $Q(x) = x^3 e^{2x} - x + x \sin 3x$

d) $Q(x) = e^x \sin x + \cos 2x$ **e)** $Q(x) = e^x + e^{2x} + x^2$ **f)** $Q(x) = xe^x - e^{3x} + 1$