A&C 1: Turing machines, computational complexity

Theory.

T1.1 Turing machine (TM).

T1.2 Variants of TMs: multi-tape, non-deterministic, one-side- or two-side-infinite tape.

T1.3 Basic notions: computation of a TM, time complexity, space complexity.

T1.4 What does it mean that two computational models are equivalent?

Exercises.

E1.1 Design a TM, which:

a) accepts the language $\{0^{2k} : k \geq 0\}$,

b) accepts the language $\{0^{2^k} : k \geq 0\}$,

c) accepts the language $\{w\overline{w} : w \in \{0,1,2\}^*\}$, where $\overline{w}$ is the word w reversed,

d) reads a string over $\{0,1,2\}$ and substitutes every 0 with 2,

e) given a string a, copies it, so that the final state of the tape is aa,

f) given a string 0^a1^b, writes the string $2^{a \cdot b}$,

What is the space and time complexity of the machine?

E1.2 Show the equivalence of one-side-infinite-tape TM and a two-side-infinite-tape TM.

E1.3 Show the equivalence of single-tape and double-tape TM.

E1.4 A TM uses a tape alphabet Σ. Show that there exists an equivalent TM with tape alphabet $\{0,1\}$.