A&C 2: Hierarchy of languages.

Theory.

T2.1 Chomsky's hierarchy of languages.

T2.2 Correspondence between automata and languages.

T2.3 Diagonal language, universal language.

T2.4 HALTING PROBLEM, undecidability.

Exercises.

E2.1 Let L_1, L_2 be recursive languages. Prove that:

a) $L_1 \cup L_2$ is recursive,

b) $L_1 \cap L_2$ is recursive,

c) $L_1 \setminus L_2$ is recursive,

d) L_1^* is recursive (by L_1^* we mean the set of all sequences obtained by concatenating words of L_1).

What happens if L_1, L_2 are recursively enumerable, but not recursive? What if one of them if recursive, and the other is recursively enumerable, but not recursive?

E2.2 Let $\mathcal{L} = \{L_1, \ldots, L_k\}$ be the set of languages over $\{0, 1, \}$, such that:

a) for each $i \leq k$, the language L_i is recursively enumerable,

b) for all i, j, such that $i \neq j$ it holds that $L_i \cap L_j = \emptyset$,

c) $\bigcup_{i=1}^{k} L_i = \{0, 1\}^*$.

Show that for each $i \leq k$, the language L_i is recursive.

E2.3 * Show that the statement from the previous exercise holds, even if $\mathcal{L}$ is infinite (but countable).

E2.4 Show that HALTING PROBLEM is undecidable.

E2.5 Consider the following computational problem, called ALL-BLUE TURING MACHINE. The input is a Turing machine M and a word w. Each state of M is either red of blue, and the initial state s_0 is blue. The computational question is whether M ever reaches a red state for input w.

Show that ALL-BLUE TURING MACHINE is undecidable.