A&C 4: Recursive functions.

Theory.

T4.1 Primitive recursive functions and predicates,

T4.2 Recursive functions, partial recursive functions,

T4.3 Relations between these classes,

T4.4 TMs and (partial) recursive functions,

T4.5 Ackerman function.

Exercises.

E4.1 Are the following functions primitive recursive?

(a) sum,

(b) multiplication,

(c) power function,

(d) factorial,

(e) predecessor,

(f) cut-off subtraction ($s(x, y) = x - y$ if $x \geq y$ and $s(x, y) = 0$ otherwise)

(g) testing whether the input is zero,

(h) Fibonacci number,

(i) remainder of the division,

(j) cut-off division ($\lfloor x/y \rfloor$)

(k) $f(x, k) = \lceil x^{1/k} \rceil$ (set $f(x, 0) = x$)

(l) comparison,

(m) divisibility (1 if x divides y and 0 otherwise)

(n) primality test,

(o) n-th prime number,

(p) Cantor indexing,

E4.2 A function f is defined as follows:

$$f(x) = \begin{cases} g_1(x) & \text{if } P_1(x), \\ g_2(x) & \text{if } P_2(x), \\ \dots & \dots \\ g_k(x) & \text{if } P_k(x), \end{cases}$$

where $g_1, \dots, g_k$ are functions are $P_1, \dots, P_k$ are predicates. Show that if $g_1, \dots, g_k, P_1, \dots, P_k$ are primitive recursive, then so is f. What happens if we have infinite number of cases?

E4.3 A function f is defined as follows:

$$\begin{cases} f(0, y) = g_1(y), \\ f(x + 1, 0) = g_2(x), \\ f(x + 1, y + 1) = h(x, y, f(x, y + 1), f(x + 1, y)). \end{cases}$$

Show that if g_1, g_2, and h are primitive recursive, then so it f.