A&C 6: Complexity theory, NP-completeness.

Theory.

T6.1 Reductions, polynomial transformations,

T6.2 Complexity classes of decidable problems: P, NP, coNP, NP-hard, NP-complete,

T6.3 Cook-Levin theorem, SAT problem.

Exercises.

E6.1 Knowing that CNF-SAT is NP-complete, prove that k-SAT is NP-complete for every $k \geq 3$.

E6.2 Show that 3-SAT remains NP-complete, even if each variable appears at most three times.

E6.3 * Show that a variant of 3-SAT, in which every variable appears exactly three times and each clause has exactly three distinct literals, is polynomially solvable.

E6.4 * Show that CLIQUE is NP-complete by a reduction from 3-SAT.

E6.5 Show that INDEPENDENT SET is NP-complete.

E6.6 Show that VERTEX COVER is NP-complete.

Problems.

CNF-SAT:

Input: a logic formula in conjunctive normal form, i.e., as a conjunction of clauses, each of which is an alternative of variables or their negations (called literals)

Question: is there an assignment of variables which makes the formula true?

k-SAT:

Input: a logic formula in conjunctive normal form, each clause contains at most k literals,

Question: is there an assignment of variables which makes the formula true?

VERTEX COVER:

Input: a graph G, an integer k,

Question: does G contain a set of k vertices, whose removal removes all edges?

CLIQUE:

Input: a graph G, an integer k,

Question: does G contain a complete subgraph of k vertices?

INDEPENDENT SET:

Input: a graph G, an integer k,

Question: does G contain an independent set of k vertices?