Knowledge Representation and Reasoning

Lecture 5:
Reasoning about Scenarios

dr Anna Maria Radzikowska
Warsaw University of Technology
Faculty of Mathematics and Information Science
E–mail: annrad@mini.pw.edu.pl

Project is co-financed by European Union within European Social Fund
Basic assumptions

In contrast to previous approaches with *branching time* model, now we will consider action domains with *linear time*.

- Inertia law.
- Linear model of time (discrete time).
- Actions with duration; during performance of the action, values of fluents changed by the actions are unknown.
- Dynamic effects of actions – one action can invoke another one(s).
- Situation can trigger actions – some states may cause executing some actions.

To represent these dynamic systems we will use action languages of the class \(AL\). For simplicity we assume that each action is performed in 1 unit of time.
As before, a *signature* is a pair \((\mathcal{F}, \mathcal{Ac})\), \(\mathcal{F} \cap \mathcal{Ac} = \emptyset\).

We have 5 types of statements:

- **fluent effect statement:**
  \[ A \text{ causes } \alpha \text{ if } \pi \]

- **action effect statement:**
  \[ A \text{ invokes } B \text{ after } d \text{ if } \pi \]

where \(A, B \in \mathcal{Ac}, \pi \in \text{Forms}(\mathcal{F})\), and \(d \in \mathbb{N}\).

Intuitively, this statement says that the action \(B\) starts after \(d\) timepoints since the action \(A\) is completed, provided that the condition \(\pi\) holds when \(A\) starts.
**Action Language \( \mathcal{AL} \) (cont.)**

- **release statement:**

\[ A \text{ releases } f \text{ if } \pi \]

- **trigger statement:**

\[ \pi \text{ triggers } A \]

Intuitively, this statement says that the action \( A \) starts at any timepoint when the condition \( \pi \) holds.

A set of statements is called a *domain description*. 
By an action scenario (scenario, for short) we mean a pair $Sc = (OBS, ACS)$, where

- $OBS$ is the set of observations: $OBS = \{(\alpha_1, t_1), \ldots, (\alpha_n, t_n)\}$, where $\alpha_i \in Forms(F)$ and $t_i \in \mathbb{N}$, $i = 1, \ldots, n$

- $ACS$ is the set of action occurrences: $ACS = \{(A_1, t_1), \ldots, (A_n, t_k)\}$, where $A_i \in Ac$ and $t_i \in \mathbb{N}$, $i = 1, \ldots, k$. 
A query in \( \mathcal{AL} \) is an expression of the form:

\[
\begin{align*}
necessary \; \alpha & \; at \; t \; when \; Sc \\
possibly \; \alpha & \; at \; t \; when \; Sc \\
necessary \; A & \; at \; t \; when \; Sc \\
possibly \; A & \; at \; t \; when \; Sc.
\end{align*}
\]

Intuitively, the first two queries state that the condition \( \alpha \) certainly holds (resp. may hold) at timepoint \( t \) when the scenario \( Sc \) is carrying out, whereas the last two queries say that the action \( A \) is certainly (resp. possibly) executed at timepoint \( t \) when the scenario \( Sc \) is carrying out.
Definition 5.1 A structure for $\mathcal{AL}$ is a system $S = (H, O, E)$ such that

- $H : \mathcal{F} \times \mathbb{N} \rightarrow \{0, 1\}$ is a history function
- $O : \mathcal{Ac} \times \mathbb{N} \rightarrow 2^\mathcal{F}$ is an occlusion function; for any action $A \in \mathcal{Ac}$ and for any timepoint $t \in \mathbb{N}$, $O(A, t)$ is the set of fluents under influence of the performance of $A$ when executed from timepoint $t - 1$ to $t$
- $E \subseteq \mathcal{Ac} \times \mathbb{N}$ is an actions occurrences relation; if $(A, t) \in E$ then $A$ occurs at timepoint $t$. We assume that for all $A, B \in \mathcal{Ac}$ and every $t \in \mathbb{N},$

$$\quad (A, t) \in E \& (B, t) \in E \implies A = B. \quad (1)$$

Note that the condition (1) guarantees that at most one action is executed at a time.
For any structure $S = (H, O, E)$ for $\mathcal{AL}$, the history function $H$ are extended for the set of all formulas according to rules well–known in propositional logic, i.e. for every timepoint $t \in \mathbb{N}$,

- $H^*(f, t) = H(f, t)$ for any $f \in \mathcal{F}$
- $H^*(\neg \alpha, t) = 1 - H^*(\alpha, t)$
- $H^*(\alpha \land \beta, t) = \min(H^*(\alpha, t), H^*(\beta, t))$
- $H^*(\alpha \lor \beta, t) = \max(H^*(\alpha, t), H^*(\beta, t))$
- $H^*(\alpha \rightarrow \beta, t) = \begin{cases} 0 & \text{iff } H^*(\alpha, t) = 1 \& H^*(\beta, t) = 0 \\ 1 & \text{otherwise} \end{cases}$
- $H^*(\alpha \equiv \beta, t) = \begin{cases} 1 & \text{iff } H^*(\alpha, t) = H^*(\beta, t) \\ 0 & \text{otherwise} \end{cases}$
Semantics of $\mathcal{AL}$ (cont.)

Let $S = (H, O, E)$ be a structure for $\mathcal{AL}$, let $Sc = (OBS, ACS)$ be a scenario, and let $D$ be a domain description. We say that $S$ is a structure for $Sc$ wrt $D$ iff

- for each observation $(\alpha, t) \in OBS$, $H(\alpha, t) = 1$
- $ACS \subseteq E$
- for each statement (always $\alpha$) $\in D$ and for each timepoint $t \in \mathbb{N}$, $H(\alpha, t) = 1$. 
Denote: \( fl(\alpha) \) – the set of fluents occurring in \( \alpha \).

- for each statement \((A \text{ causes } \alpha \text{ if } \pi) \) \( \in D \) and for each timepoint \( t \in \mathbb{N} \), if \( H(\pi, t) = 1 \) and \((A, t) \in E\), then \( H(\alpha, t + 1) = 1 \) and \( fl(\alpha) \subseteq O(A, t + 1) \)
- for each statement \((A \text{ releases } f \text{ if } \pi) \) \( \in D \) and for each timepoint \( t \in \mathbb{N} \), if \( H(\pi, t) = 1 \) and \((A, t) \in E\), then \( f \in O(A, t + 1) \)
- for each statement \((\pi \text{ triggers } A) \) \( \in D \) and for each timepoint \( t \in \mathbb{N} \), if \( H(\pi, t) = 1 \), then \((A, t) \in E\)
- for each statement \((A \text{ invokes } B \text{ after } d \text{ if } \pi) \) \( \in D \) and for each timepoint \( t \in \mathbb{N} \), if \( H(\pi, t) = 1 \) and \((A, t) \in A\), then \((B, t + d + 1) \in E\).
Observe:

- Any change (in fluents’ values) are allowed **ONLY** in occlusion regions.
- Consequently, we will be interested in structures $S = (H, O, E)$ for $Sc = (OBS, ACS)$ wrt $D$, which occlusion functions $O$ determine the smallest occlusion regions.
Denote

Let $O_1, O_2 : X \to 2^Y$. We write

- $O_1 \preceq O_2$ iff $O_1(x) \subseteq O_2(x)$ for every $x \in X$.
- $O_1 \prec O_2$ iff $O_1 \preceq O_2$ and $O_1 \neq O_2$.

**Definition 5.2** Let $S = (H, O, E)$ be a structure for a scenario $Sc = (OBS, ACS)$ wrt a domain description $D$. We say that $S$ is $O$-minimal iff there is no structure $S' = (H', O', E')$ for $Sc$ wrt $D$ such that $O' \prec O$. 
Definition 5.3 Let \( S = (H, O, E) \) be a structure for a scenario \( Sc = (OBS, ACS) \) wrt a domain description \( D \). We say that \( S \) is a model of \( Sc \) wrt \( D \) iff

\[(M.1) \quad S \text{ is } O\text{-minimal}\]

\[(M.2) \quad \text{for every timepoint } t \in \mathbb{N}, \quad \{ f \in F : H(f, t) \neq H(f, t + 1) \} \subseteq O(A, t + 1) \quad \text{for some action } A \in Ac \]

\[(M.3) \quad \text{there is no structure } S' = (H', O', E') \text{ for } Sc \text{ wrt } D \text{ satisfying (M.1)–(M.2) such that } E' \subset E. \]
We say that a scenario $Sc$ is \textit{consistent} wrt a domain description $D$ iff there exists a model $S$ of $Sc$ wrt $D$; otherwise it is called \textit{inconsistent}.
Example 5.1: Inconsistency

Let $D$ be a domain description with two actions, $A$ and $B$, and let a scenario $Sc = (OBS, ACS)$ be given as

- $OBS = \emptyset$
- $ACS = \{(A, 1), (B, 1)\}$.

Since $A$ and $B$ are to be executed parallel, $Sc$ is inconsistent wrt any $D$ (with the actions $A$ and $B$) – there is no structure for $Sc$ wrt any $D$. 
Example 5.2: (In)consistency

Consider the following domain description $D$ and the following two scenarios $S_{c_1} = (OBS_1, ACS_1)$ and $S_{c_2} = (OBS_2, ACS_2)$:

- $A$ causes $f$;
- $B$ causes $\neg f$;
- $C$ causes $g$;
- $A$ invokes $C$ after 1.

- $OBS_1 = OBS_2 = \emptyset$
- $ACS_1 = \{(A, 1), (B, 3)\}$
- $ACS_2 = \{(A, 1), (B, 2)\}$.

For any $S = (H, O, E)$ for $S_{c_1}$ wrt $D$, we have $(A, 1), (B, 3) \in E$. Since $(C, 3) \in E$, $S_{c_1}$ is inconsistent wrt $D$.

For any $S = (H, O, E)$ for $S_{c_2}$ wrt $D$, $(A, 1), (B, 2), (C, 3) \in E$, so no inconsistency occurs.
Example 5.3: Modification of YSP

Consider the following domain description $D$:

- **Load** causes $\text{loaded}$;
- **Shoot** causes $\neg \text{loaded}$;
- **Shoot** causes $\neg \text{alive}$ if $\text{loaded} \land \neg \text{hidden}$;
- **Load** invokes **Escape**;
- **Escape** releases $\text{hidden}$.

and a scenario $Sc = (\text{OBS}, \text{ACS})$, where

- $\text{OBS} = \{(\text{alive} \land \neg \text{loaded} \land \neg \text{hidden}, 0)\}$
- $\text{ACS} = \{(\text{Load}, 1), (\text{Shoot}, 3)\}$.
There are two main classes of structures for $Sc$ wrt $D$. Namely,

**Class 1:**

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 & 4 \\
\text{Load} & \text{Escape} & \text{Shoot} \\
\ \ a & a? & a? & a? & a? \\
\neg l & l? & l^* & l? & \neg l^* \\
\neg h & h? & h? & h^* & h?
\end{array}
\]

\[
\{l\} \subseteq Occlude(\text{LOAD}, 2)
\]

\[
\{h\} \subseteq Occlude(\text{ESCAPE}, 3)
\]

\[
\{l\} \subseteq Occlude(\text{SHOOT}, 4).
\]

\[
\text{Occurrences of actions: } \{(\text{LOAD}, 1), (\text{ESCAPE}, 2), (\text{SHOOT}, 3)\} \subseteq E.
\]
Structures for $\mathcal{Sc}$ wrt $\mathcal{D}$ (cont.)

Class 2:

\begin{itemize}
  \item \{l\} ⊆ Occlude(LOAD, 2)
  \item Occlusion regions: \{h\} ⊆ Occlude(ESCAPE, 3)
  \item \{l\} ⊆ Occlude(SHOOT, 4).
  \item Occurrences of actions: \{(LOAD, 1), (ESCAPE, 2), (SHOOT, 3)\} ⊆ E.
\end{itemize}
Class 2 has two subclasses:

**Subclass 1:**

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a?</td>
<td>a?</td>
<td>a?</td>
<td>¬a*</td>
</tr>
<tr>
<td>¬l</td>
<td>l?</td>
<td>l*</td>
<td>l</td>
<td>¬l*</td>
</tr>
<tr>
<td>¬h</td>
<td>h?</td>
<td>h?</td>
<td>¬h*</td>
<td>h?</td>
</tr>
</tbody>
</table>

**Occlusion regions:**

\[\{l\} \subseteq \text{Occlude}(\text{LOAD}, 2)\]
\[\{h\} \subseteq \text{Occlude}(\text{ESCAPE}, 3)\]
\[\{a, l\} \subseteq \text{Occlude}(\text{SHOOT}, 4)\].

**Occurrences of actions:**

\[\{(\text{LOAD}, 1), (\text{ESCAPE}, 2), (\text{SHOOT}, 3)\} \subseteq E\].

Subclass 2:

<table>
<thead>
<tr>
<th></th>
<th>Load</th>
<th>Escape</th>
<th>Shoot</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>a</td>
<td>a?</td>
<td>a?</td>
</tr>
<tr>
<td>1</td>
<td>a?</td>
<td>a?</td>
<td>a?</td>
</tr>
<tr>
<td>2</td>
<td>l</td>
<td>l*</td>
<td>¬l</td>
</tr>
<tr>
<td>3</td>
<td>l*</td>
<td>¬l</td>
<td>¬l*</td>
</tr>
<tr>
<td>4</td>
<td>h</td>
<td>h?</td>
<td>¬h*</td>
</tr>
<tr>
<td></td>
<td>¬h</td>
<td>h?</td>
<td>h?</td>
</tr>
</tbody>
</table>

Occlusion regions:

\[
\{l\} \subseteq Occlude(\text{LOAD}, 2) \quad \{h\} \subseteq Occlude(\text{ESCAPE}, 3) \quad \{l\} \subseteq Occlude(\text{SHOOT}, 4).
\]

Occurrences of actions:

\[
\{(\text{LOAD}, 1), (\text{ESCAPE}, 2), (\text{SHOOT}, 3)\} \subseteq E.
\]
Two main classes of $O$–minimal structures for $Sc$ wrt $D$:

**Class 1:**

<table>
<thead>
<tr>
<th>Time</th>
<th>Load</th>
<th>Escape</th>
<th>Shoot</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$a$</td>
<td>$a?$</td>
<td>$a?$</td>
</tr>
<tr>
<td>1</td>
<td>$a?$</td>
<td>$a?$</td>
<td>$a?$</td>
</tr>
<tr>
<td>2</td>
<td>$\neg l$</td>
<td>$l*$</td>
<td>$l?$</td>
</tr>
<tr>
<td>3</td>
<td>$\neg h$</td>
<td>$h?$</td>
<td>$h*$</td>
</tr>
<tr>
<td>4</td>
<td>$\neg l*$</td>
<td>$h?$</td>
<td>$h?$</td>
</tr>
</tbody>
</table>

- Occlusion regions:
  - $Occlude(\text{LOAD}, 2) = \{l\}$
  - $Occlude(\text{ESCAPE}, 3) = \{h\}$
  - $Occlude(\text{SHOOT}, 4) = \{l\}$.

- Occurrences of actions: $\{(\text{LOAD}, 1), (\text{ESCAPE}, 2), (\text{SHOOT}, 3)\} \subseteq E$. 
Structures for $Sc$ wrt $D$ (cont.)

And two subclasses of the $2^{nd}$ class:

1$st$ subclass:

\[
\begin{array}{ccccccc}
\text{0} & \text{1} & \text{2} & \text{3} & \text{4} \\
0 & 1 & 2 & 3 & 4 \\
\text{a} & \text{a?} & \text{a?} & \text{a?} & \neg a* \\
\neg l & l? & l* & l & \neg l* \\
\neg h & h? & h? & \neg h* & h? \\
\end{array}
\]

\[
\text{Occlude}(\text{LOAD}, 2) = \{l\}
\]

\[
\text{Occlusion regions: } \text{Occlude}(\text{ESCAPE}, 3) = \{h\}
\]

\[
\text{Occlude}(\text{SHOOT}, 4) = \{a, l\}.
\]

\[
\text{Occurrences of actions: } \{(\text{LOAD}, 1), (\text{ESCAPE}, 2), (\text{SHOOT}, 3)\} \subseteq E.
\]
2nd subclass:

\[ 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \]

- **Load**
  - 0: \( a \)
  - 1: \( a? \)

- **Escape**
  - 2: \( a? \)
  - 3: \( a? \)

- **Shoot**
  - 4: \( a? \)

\[ \neg l \quad l? \quad l^* \quad \neg l \quad \neg l^* \]

\[ \neg h \quad h? \quad h? \quad \neg h^* \quad h? \]

\[ \text{Occlusion regions:} \quad \text{Occlude}(\text{Load}, 2) = \{ l \} \]

\[ \text{Occlude}(\text{Escape}, 3) = \{ h \} \]

\[ \text{Occlude}(\text{Shoot}, 4) = \{ l \}. \]

\[ \text{Occurrences of actions:} \quad \{(\text{Load}, 1), (\text{Escape}, 2), (\text{Shoot}, 3)\} \subseteq E. \]
There are two models of $Sc$ wrt $D$:

1\textsuperscript{st} model $S_1$:

\begin{itemize}
  \item \textit{Occlusion regions:} $Occlude(LOAD, 2) = \{l\}$
  \item $Occlude(ESCAPE, 3) = \{h\}$
  \item $Occlude(SHOOT, 4) = \{l\}$.
  \item Occurrences of actions: $\{(LOAD, 1), (ESCAPE, 2), (SHOOT, 3)\} = E$.
\end{itemize}
Models of $S_c \text{ wrt } D$

$2^{nd}$ model $S_2$: 

<table>
<thead>
<tr>
<th></th>
<th>Load</th>
<th>Escape</th>
<th>Shoot</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$a$</td>
<td>$a$</td>
<td>$a$</td>
</tr>
<tr>
<td>1</td>
<td>$a$</td>
<td>$a$</td>
<td>$a$</td>
</tr>
<tr>
<td>2</td>
<td>$a$</td>
<td>$l^*$</td>
<td>$l$</td>
</tr>
<tr>
<td>3</td>
<td>$a$</td>
<td>$l^*$</td>
<td>$l^*$</td>
</tr>
<tr>
<td>4</td>
<td>$a$</td>
<td>$l^*$</td>
<td>$l^*$</td>
</tr>
</tbody>
</table>

Occlusion regions: 
- $\text{Occlude}(\text{LOAD}, 2) = \{l\}$
- $\text{Occlude}(\text{ESCAPE}, 3) = \{h\}$
- $\text{Occlude}(\text{SHOOT}, 4) = \{a, l\}$. 

Occurrences of actions: 
- $\{\text{(LOAD, 1)}, \text{(ESCAPE, 2)}, \text{(SHOOT, 3)}\} = E$. 
Let $Sc$ be a scenario and let $D$ be a domain description. We say that a query $Q$ is a

**consequence of $Sc$ wrt $D$**, in symbols $D, Sc \models Q$, iff

- if $Q$ is of the form $\alpha$ at $t$ when $Sc$, then for every model $S = (H, O, E)$ of $Sc$ wrt $D$, it holds $H(\alpha, t) = 1$

- if $Q$ is of the form $A$ at $t$ when $Sc$, then for every model $S = (H, O, E)$ of $Sc$ wrt $D$, it holds $(A, t) \in E$. 

Example 5.3 (cont.)

\[
\begin{aligned}
&\text{Load} & \text{Escape} & \text{Shoot} \\
0 & a & a & a & a & a \\
1 & a & a & a & a & a \\
2 & a & l^* & l & l & \neg l^* \\
3 & a & l & l & h^* & h \\
4 & a & l & l & h & \neg h \\
\end{aligned}
\]

\[
\begin{aligned}
&\text{Load} & \text{Escape} & \text{Shoot} \\
0 & a & a & a & a & a \\
1 & a & a & a & a & a \\
2 & a & l^* & l & l & \neg l^* \\
3 & a & l & l & h^* & h \\
4 & a & l & l & h & \neg h \\
\end{aligned}
\]

\[Sc, D \models \neg \text{loaded at } t \text{ when } Sc \text{ for } t \geq 4\]

\[Sc, D \models \text{ESCAPE at } 2 \text{ when } Sc\]

\[Sc, D \not\models \text{alive at } 4 \text{ when } Sc\]

\[Sc, D \not\models \neg \text{alive at } 4 \text{ when } Sc.\]
Thank you for your attention!