The work of Stanisław Janeczko

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Singularities of functions on a manifold with boundary

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Collaborators

Long Van Cao, Wojciech Domitrz, Takuo Fukuda, Peter J. Giblin, Goo Ishikawa, Shyuichi Izumiya, Zbigniew Jelonek, Jacek Komorowski, Adam Kowalczyk, Małgorzata Mikosz, Tadeusz Mostowski, Zbigniew Pasternak-Winiarski, Fernand Pelletier, Galina Plotnikova, Richard Mark Roberts, Ian N. Stewart, Eligiusz Wajnryb, Mariusz Zając, Michail Zhitomirskii, Henryk Żołądek

Local symplectic algebra (V. I. Arnol'd,'99)

$$A_{2k}: t \mapsto (t^2, t^{2k+1}, 0, \cdots, 0)$$

Theorem (V. I. Arnold,1999)

A germ of the type A_{2k} is symplectically equivalent to one and only one of the following germs

$$A_{2k,0}: \ t \mapsto (t^2, t^{2k+1}, 0, \cdots, 0),$$

$$A_{2k,r}: \ t \mapsto (t^2, t^{2k+1+2r}, t^{2k+1}, 0, \cdots, 0) \text{ for } r = 1, \cdots, 2k,$$

where $(p_1, q_1, \cdots, p_n, q_n)$ is a coordinate system on $\mathbb{C}^{2n}$ and $\omega = \sum_{i=1}^n dp_i \wedge dq_i$.

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Algebraic restrictions

Let ω_1, ω_2 be germs of smooth differential k-forms on a smooth manifold M. Let N be the germ of a subset of M.

Definition

ω_1 and ω_2 have the same algebraic restriction to N if there exist the germ of a smooth diffential k-form α vanishing at points of N and the germ of a smooth diffrential $(k-1)$-form β vanishing at points of N such that

$$\omega_1 - \omega_2 = \alpha + d\beta.$$

The algebraic restriction of ω to N is denoted by $[\omega]_N$.

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The relative Darboux theorem for quasi-homogeneous substes

Theorem (W.D., S. Janeczko, M. Zhitomirskii)

Germs of quasi-homogeneous subsets N_1, N_2 of a smooth symplectic manifold (M, ω) are symplectomorphic if and only if their algebraic restrictions $[\omega]_{N_1}$ i $[\omega]_{N_2}$ are diffeomorphic i.e. there exists a diffeomorphism-germ $\Psi : M \to M$ such that $\Psi(N_1) = N_2$ and $[\Psi^*\omega]_{N_1} = [\omega]_{N_1}$.

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Geometric interpretation of zero algebraic restriction

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The germ of a quasi-homogeneous subset N of a symplectic space $(\mathbb{R}^{2n}, \omega)$ is contained in a non-singular Lagrangian submanifold if and only if the germ of the symplectic form ω has zero algebraic restriction to N.

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Discrete symplectic invariants.

Definition

Let N be the germ of a subset of a symplectic space $(\mathbb{R}^{2n}, \omega)$. The isotropy index of N is the maximal order of vanishing of the germs of 2-forms $\omega|_{TM}$ over all non-singular submanifolds M containing N.

Definition

Let N be the germ of a variety in a symplectic space $(\mathbb{R}^{2n}, \omega)$. Let (N) be the orbit of N with respect to the group of local diffeomorphisms and let $(N)^{\mathrm{symp}}$ be the orbit of N with respect to the group of local symplectomorphisms. The symplectic multiplicity of N is the codimension of $(N)^{\mathrm{symp}}$ in (N).

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Symplectic $A-D$ singularities

$$N^i = \left\{H(p_1,p_2) = q_1 - \int_0^{p_2} F_i(p_1,t)dt = q_{\geq 2} = p_{\geq 3} = 0\right\} \subset \left(\mathbb{R}^{2n}, \omega_0\right),$$

$H(x_1,x_2)$	$F_i(x_1,x_2)$ for $i=0,1,\ldots,\mu$
$A_k: x_1^{k+1} - x_2^2,\ k\geq 1$	$F_0=1,\ F_i=x_1^i$ for $i=1,\ldots,k-1,\ F_k=0$
$D_k: x_1^2x_2 - x_2^{k-1},\ k\geq 4$	$F_0=1,\ F_i=bx_1+x_2^i$ for $i=1,\ldots,k-4$, $F_{k-3}=(\pm 1)^k x_1 + bx_2^{k-3}$, $F_{k-2}=x_2^{k-3},\ F_{k-1}=x_2^{k-2},\ F_k=0$

symplectic $E_6 - E_7$ singularitia

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$H(x_1,x_2)$	$F_i(x_1,x_2),\ i = 0,1,\dots,\mu$
$E_6 : x_1^3 - x_2^4$	$F_0 = 1,\ F_1 = \pm x_2 + bx_1,\ F_2 = x_1 + bx_2^2,$ $F_3 = x_2^2 + bx_1x_2,\ F_4 = \pm x_1x_2,\ F_5 = x_1x_2^2,\ F_6 = 0$
$E_7 : x_1^3 - x_1x_2^3$	$F_0 = 1,\ F_1 = x_2 + bx_1,\ F_2 = \pm x_1 + bx_2^2,$ $F_3 = x_2^2 + bx_1x_2,\ F_4 = \pm x_1x_2 + bx_2^3,$ $F_5 = x_2^3,\ F_6 = x_2^4,\ F_7 = 0$

Symplectic E_8 singularity

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$H(x_1, x_2)$	$F_i(x_1, x_2),\ i = 0, 1, \ldots, \mu$
$E_8 : x_1^3 - x_2^5$	$F_0 = \pm 1,\ F_1 = x_2 + bx_1,\ F_2 = x_1 + b_1 x_2^2 + b_2 x_2^3,$ $F_3 = \pm x_2^2 + bx_1 x_2,\ F_4 = \pm x_1 x_2 + bx_2^3,$ $F_5 = x_2^3 + bx_1 x_2^2,\ F_6 = x_1 x_2^2,\ F_7 = \pm x_1 x_2^3,$ $F_8 = 0$

Generic sigularities of symplectic and quasi-symplectic immersions

$$\left(M^{2n}, \omega, S_1^k \cup S_2^k\right)_p, \tag{1}$$

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The **linearization** of $\left(M^{2n},\ \omega,\ S_1^k \cup S_2^k\right)_p$ is the following tuple

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Theorem (W.D., S. Janeczko, M. Zhitomirskii)

Let $2 \leq k \leq n$. In the problem of classifying germs of immersed k-dimensional submanifolds of a symplectic $2n$-manifold at a double point satisfying (G1)-(G6) *the tuple of characteristic numbers is a complete invariant.*

Tne characteristic Hamiltonians

$n < k \leq 2n - 2$

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We obtain $s = s(k, 2n)$ function germs:

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Happy birthday Staszek!