Historia algorytmu Fast Fourier Transform

"The most important numerical algorithm of our lifetime"

~ Gilbert Strang

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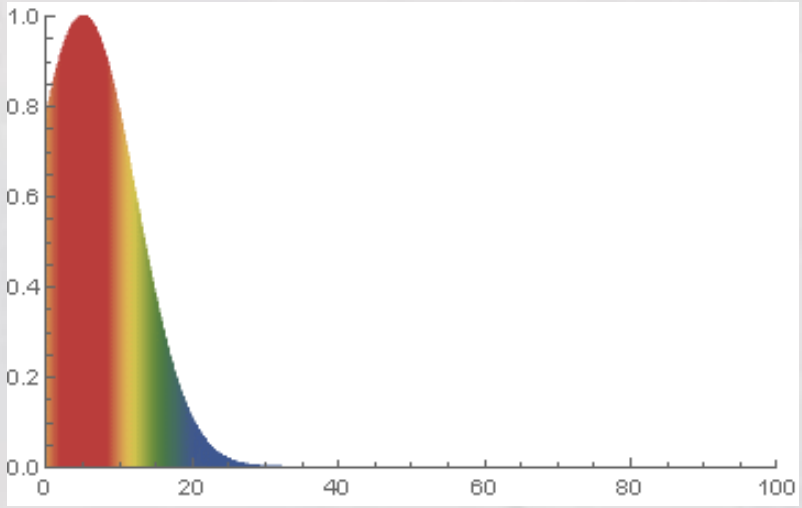
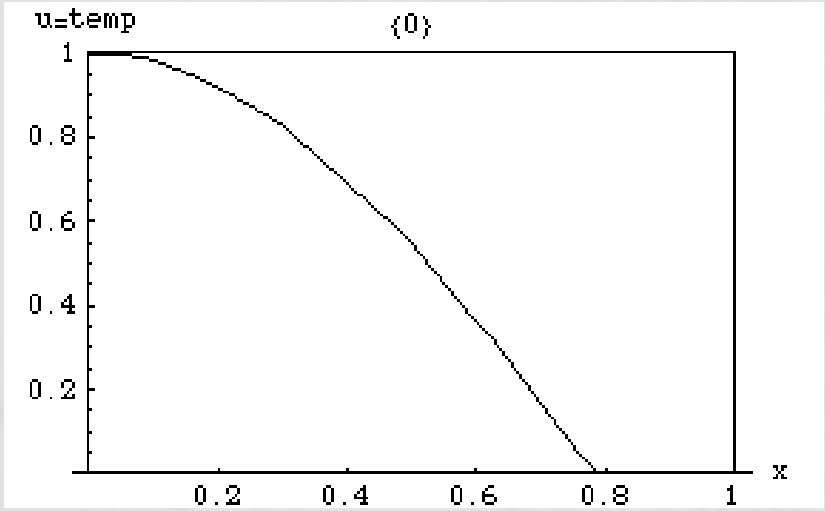
Jean Baptiste Joseph Fourier (1768 – 1830)

Równanie przewodnictwa cieplnego

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$





Szereg Fouriera

$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{2\pi}{T} nx\right) + b_n \sin\left(\frac{2\pi}{T} nx\right) \right]$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cos\left(\frac{2\pi}{T} nx\right) dx, \quad n = 0, 1, 2, \ldots,$$

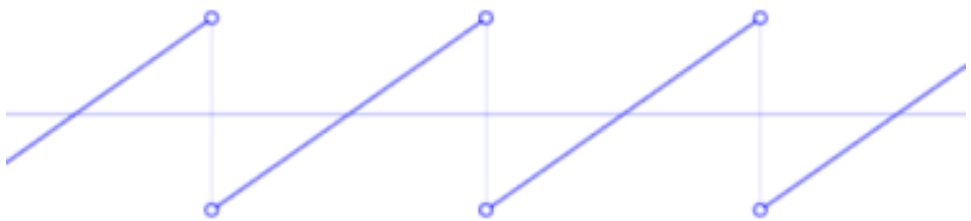
$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin\left(\frac{2\pi}{T} nx\right) dx, \quad n = 1, 2, 3, \ldots$$

Szereg Fouriera

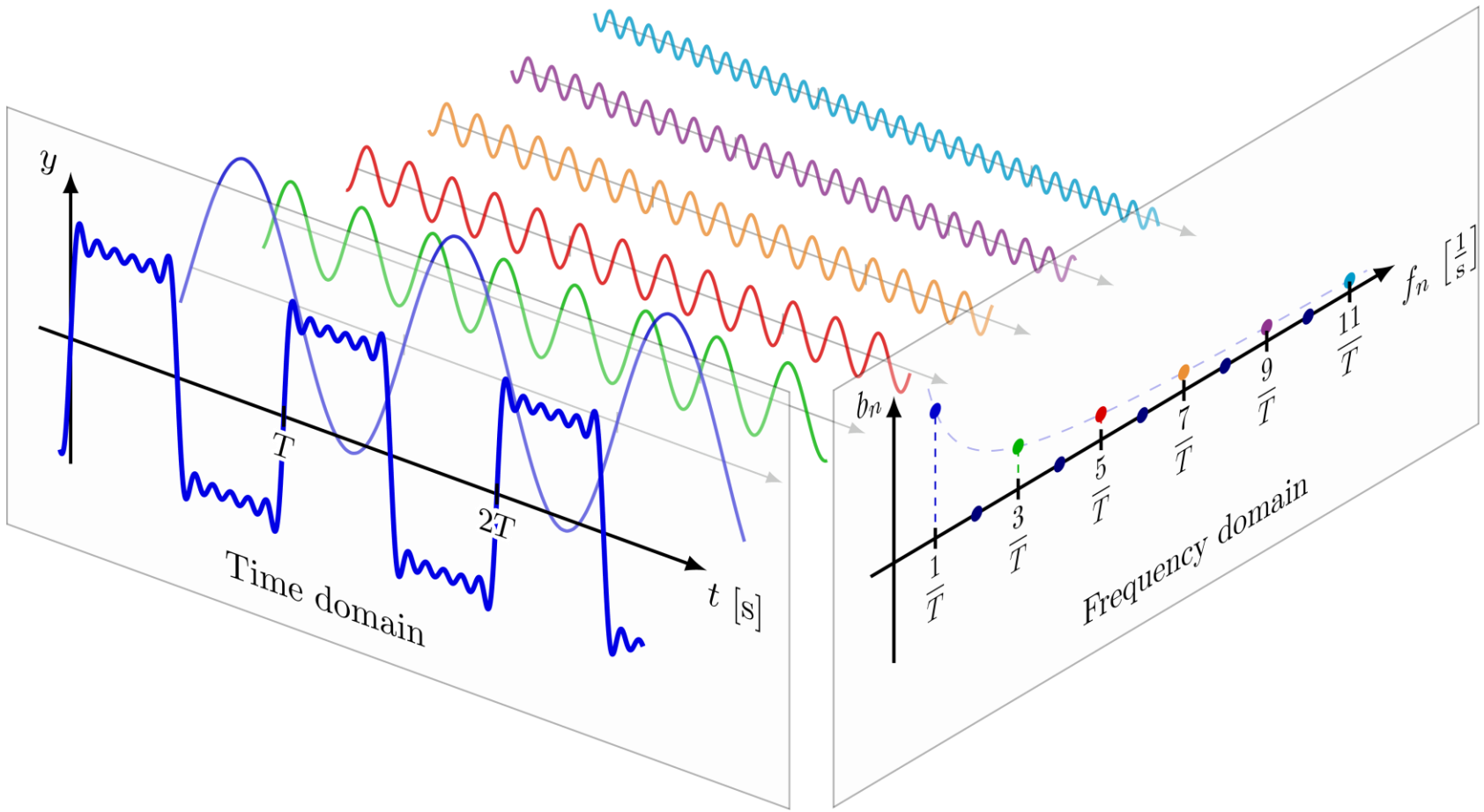
$$S(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega x}$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(x) e^{-in\omega x} dx$$

N = 0



N = 0


y
T
2T
t [s]
Time domain
b_n
$\frac{1}{T}$
$\frac{3}{T}$
$\frac{5}{T}$
$\frac{7}{T}$
$\frac{9}{T}$
$\frac{11}{T}$
f_n $\left[\frac{1}{s}\right]$
Frequency domain

Transformacja Fouriera

$$\hat{f}\,(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\,dt$$

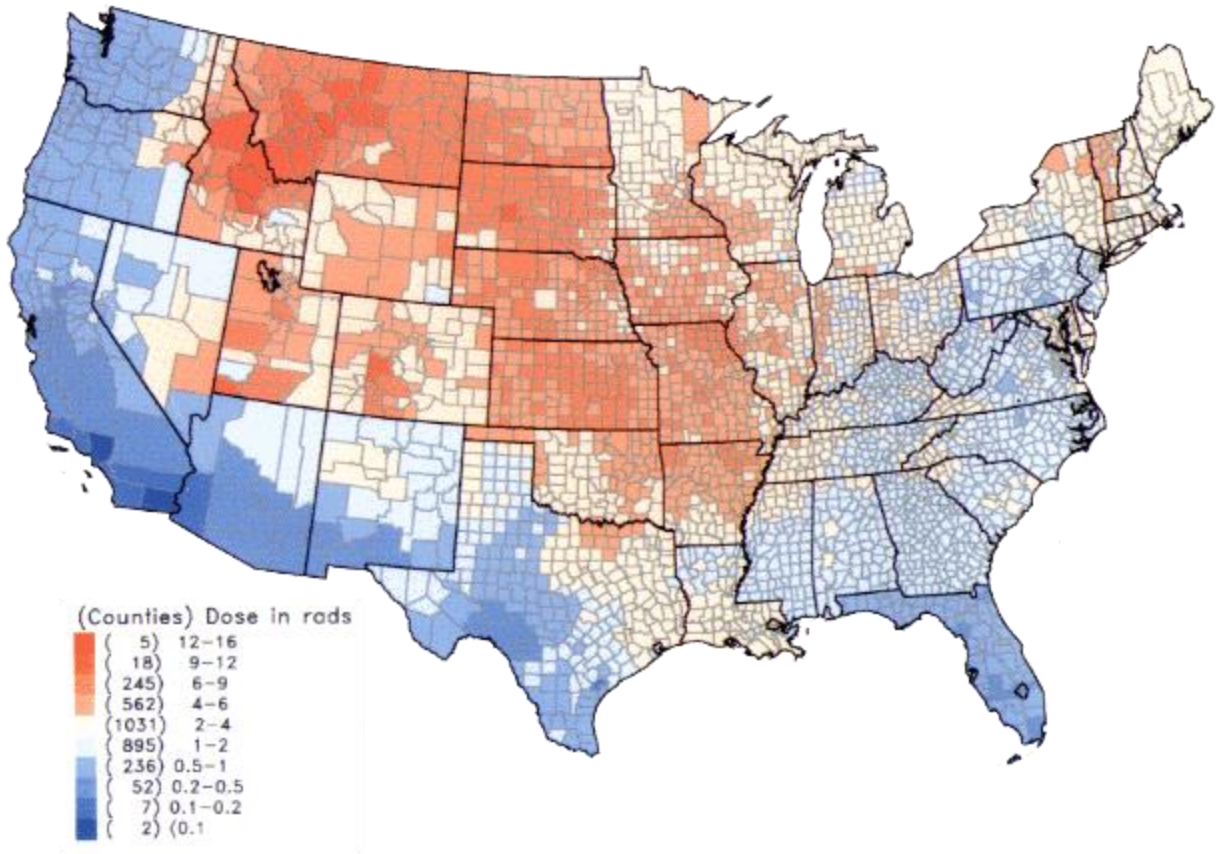
$$= \int_{-\infty}^{\infty} f(t)\cos(\omega t)\,dt - i\int_{-\infty}^{\infty} f(t)\sin(\omega t)\,dt$$

$f(x)$

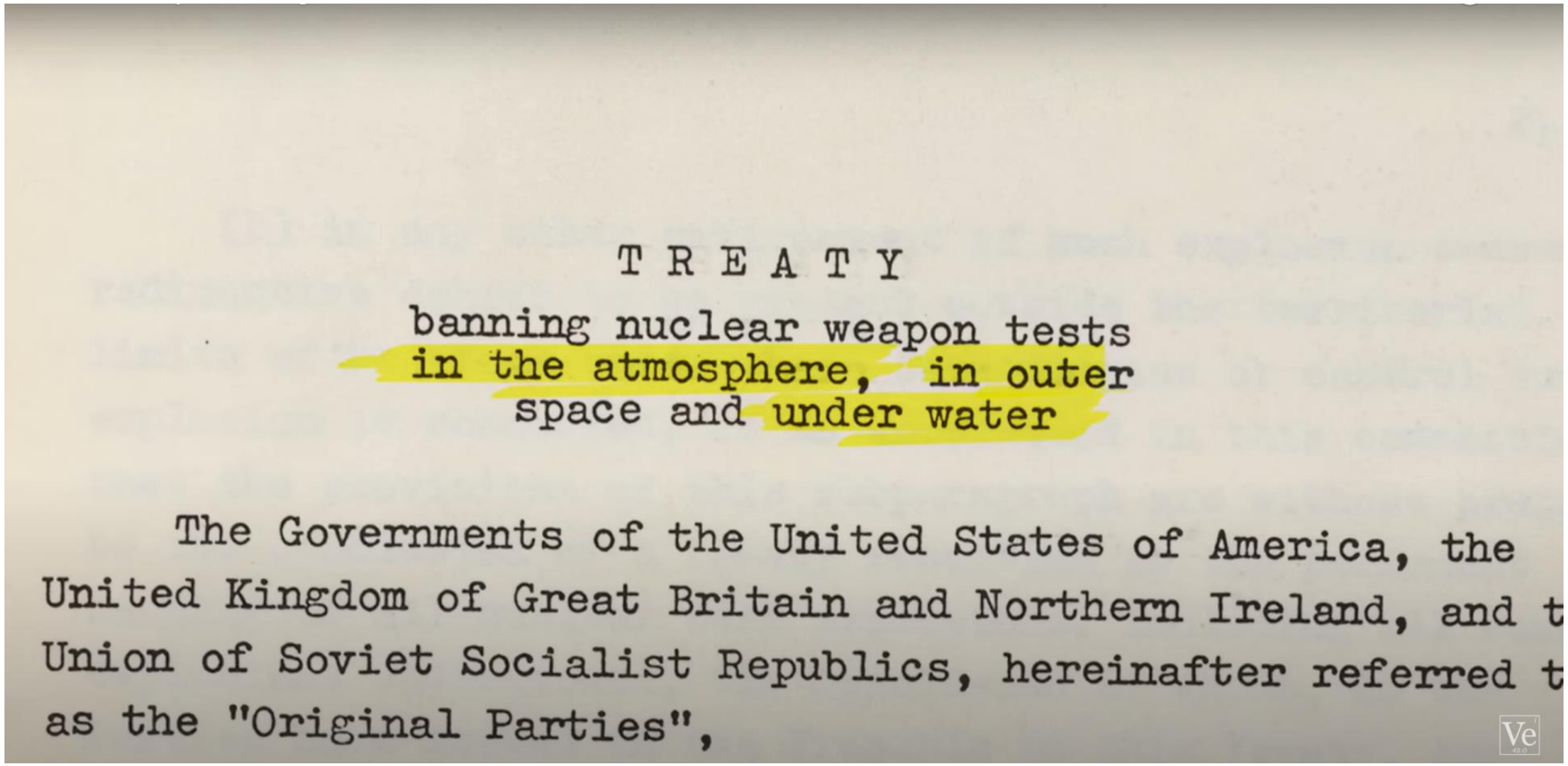
Animacja

Tło historyczne FFT

Problemy z testowaniem bomb



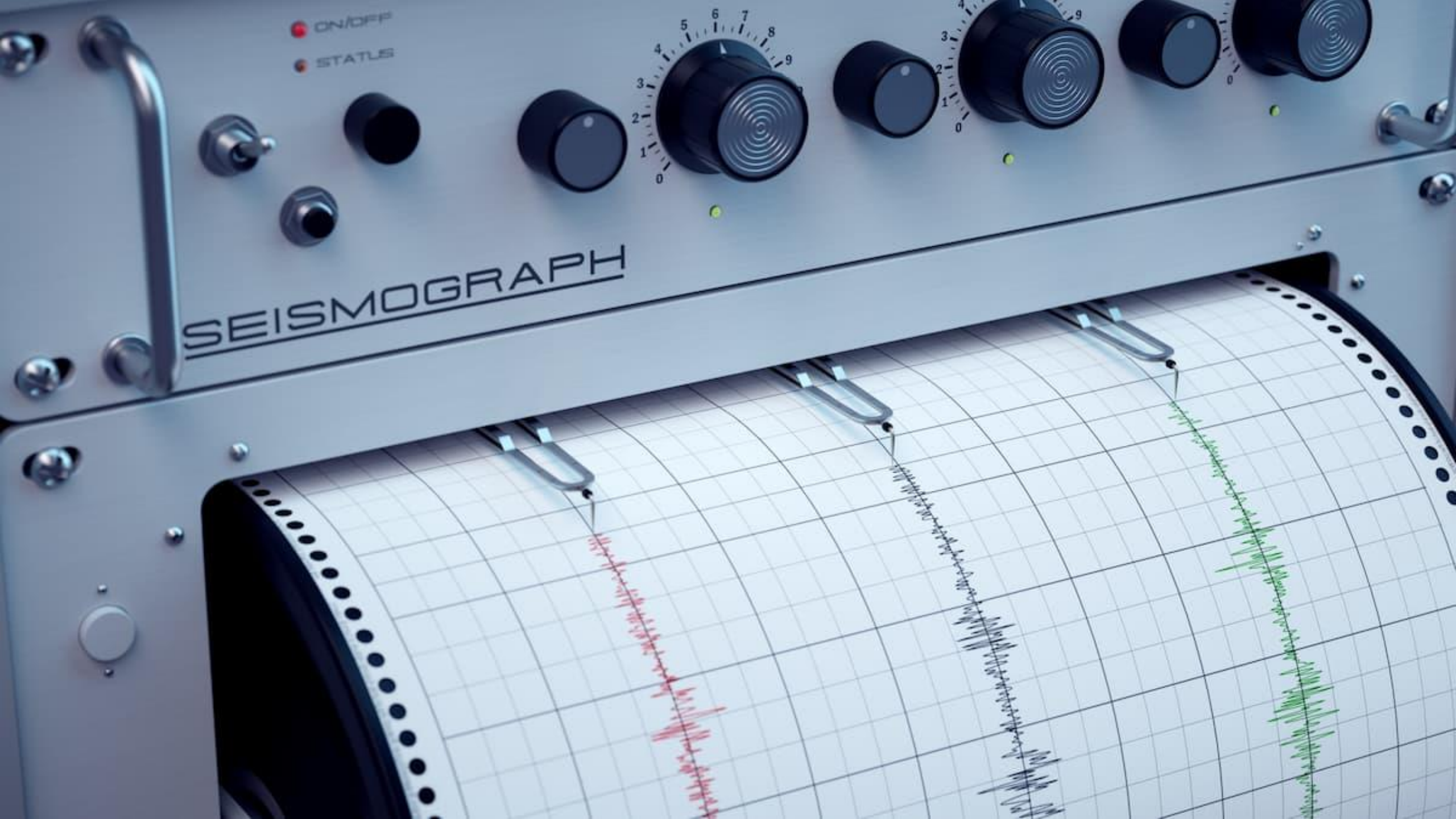
Problemy z testowaniem bomb



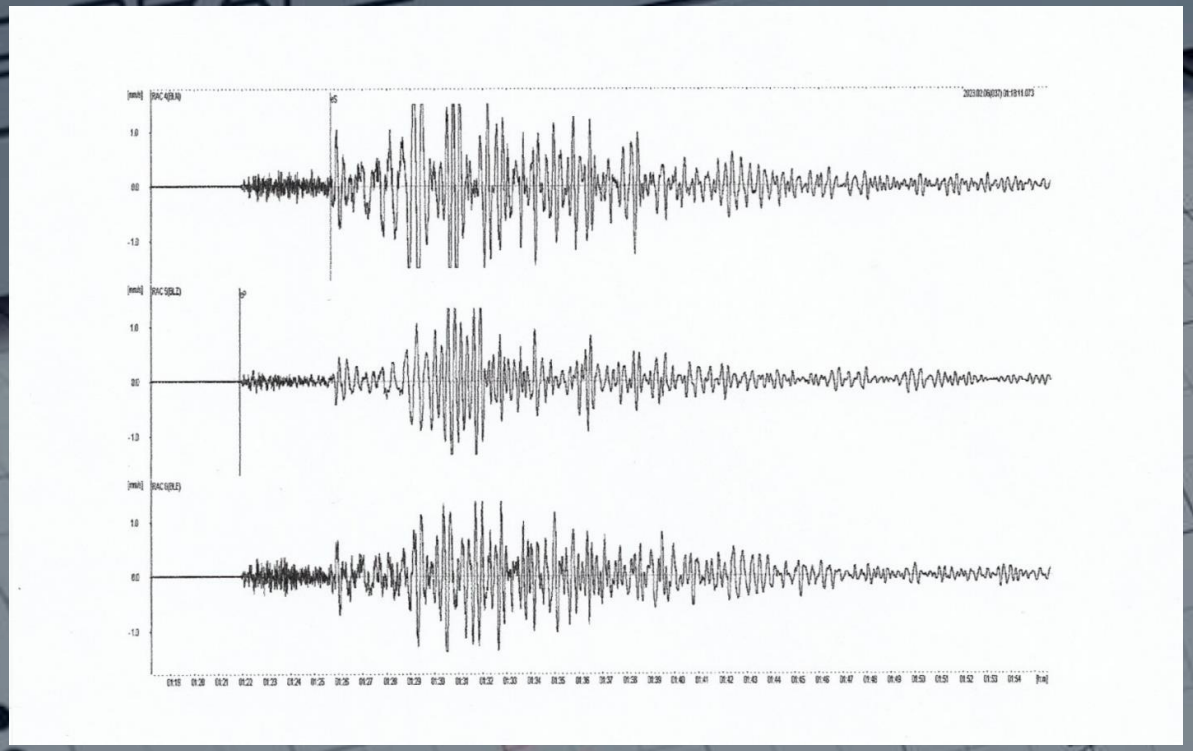
T R E A T Y

banning nuclear weapon tests
in the atmosphere, in outer
space and under water

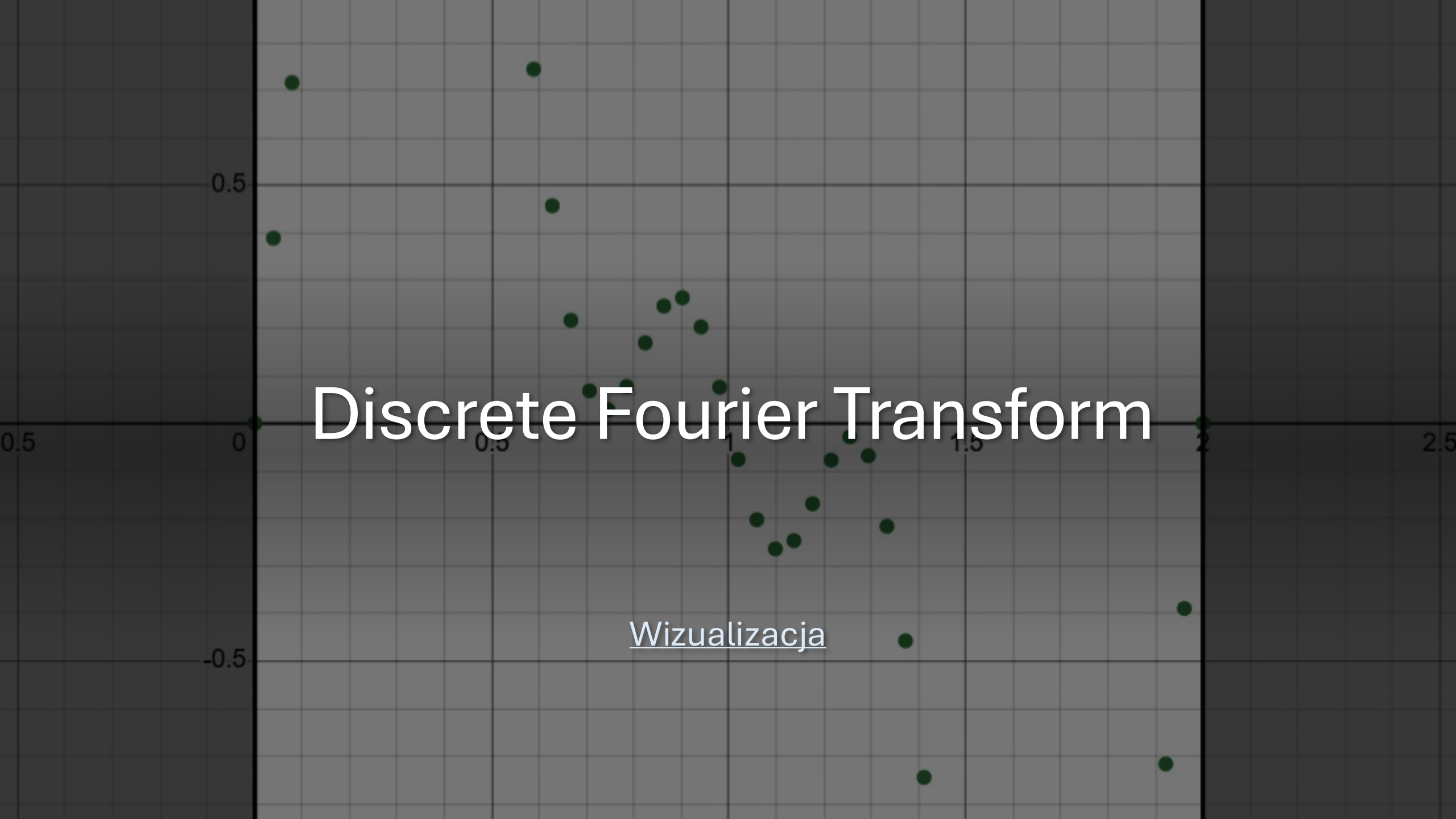
The Governments of the United States of America, the United Kingdom of Great Britain and Northern Ireland, and t Union of Soviet Socialist Republics, hereinafter referred t as the "Original Parties",


ON/OFF
STATUS
SEISMOGRAPH

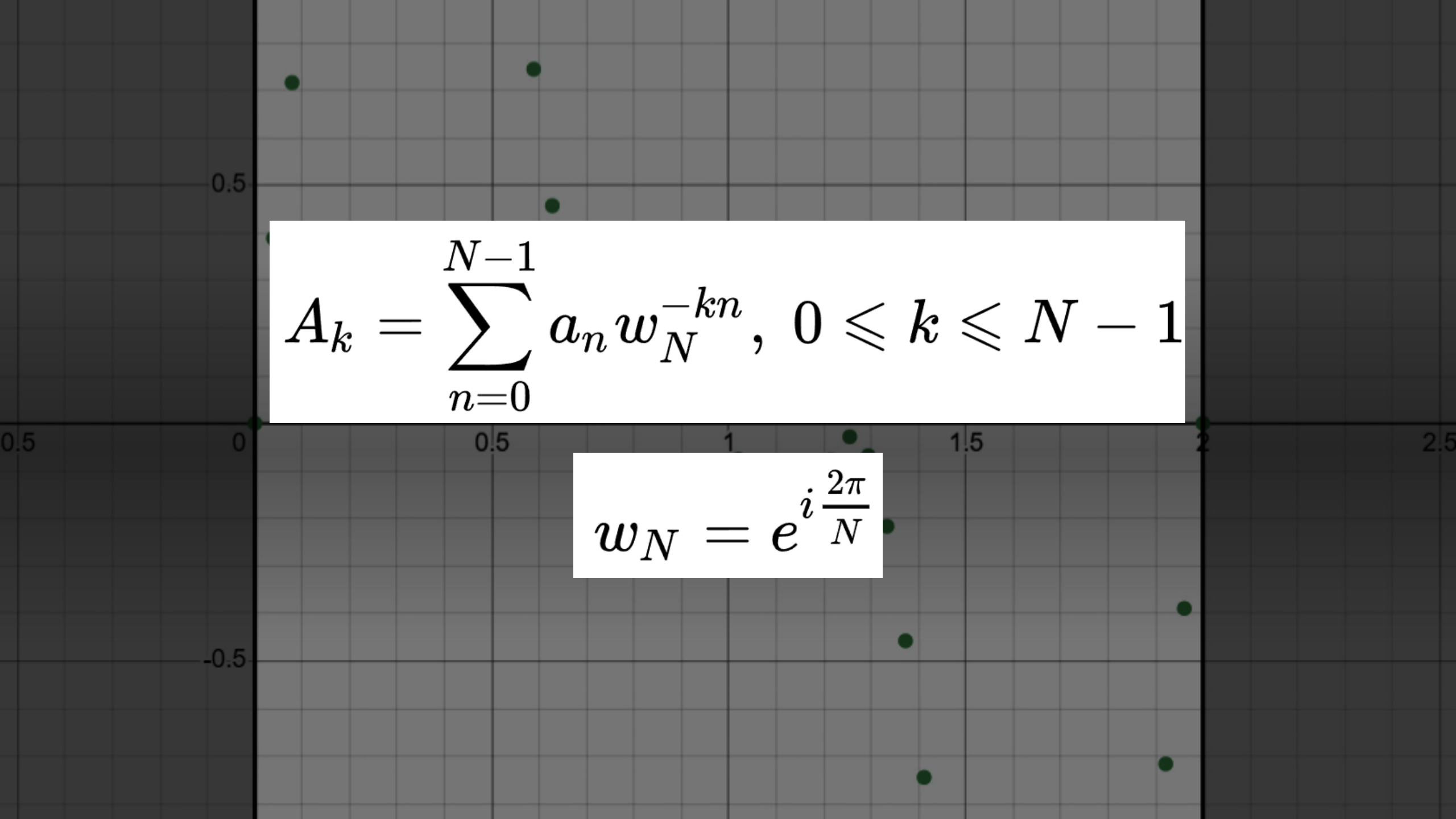
Jak odróżnić test bomby atomowej od trzęsienia ziemi?



Discrete Fourier Transform



Wizualizacja

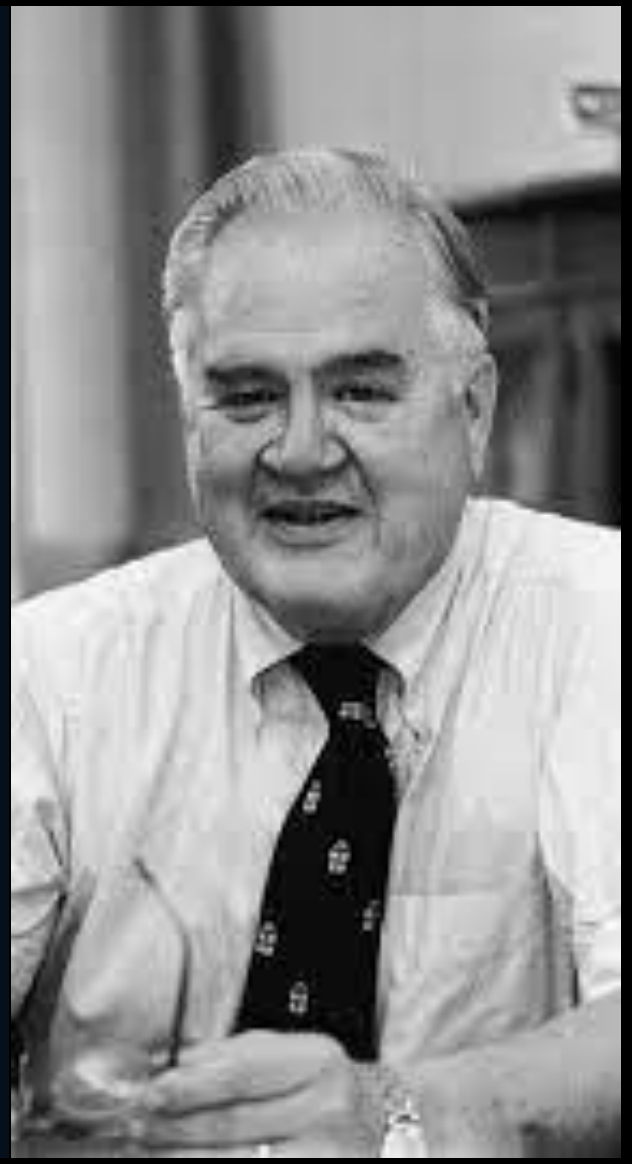


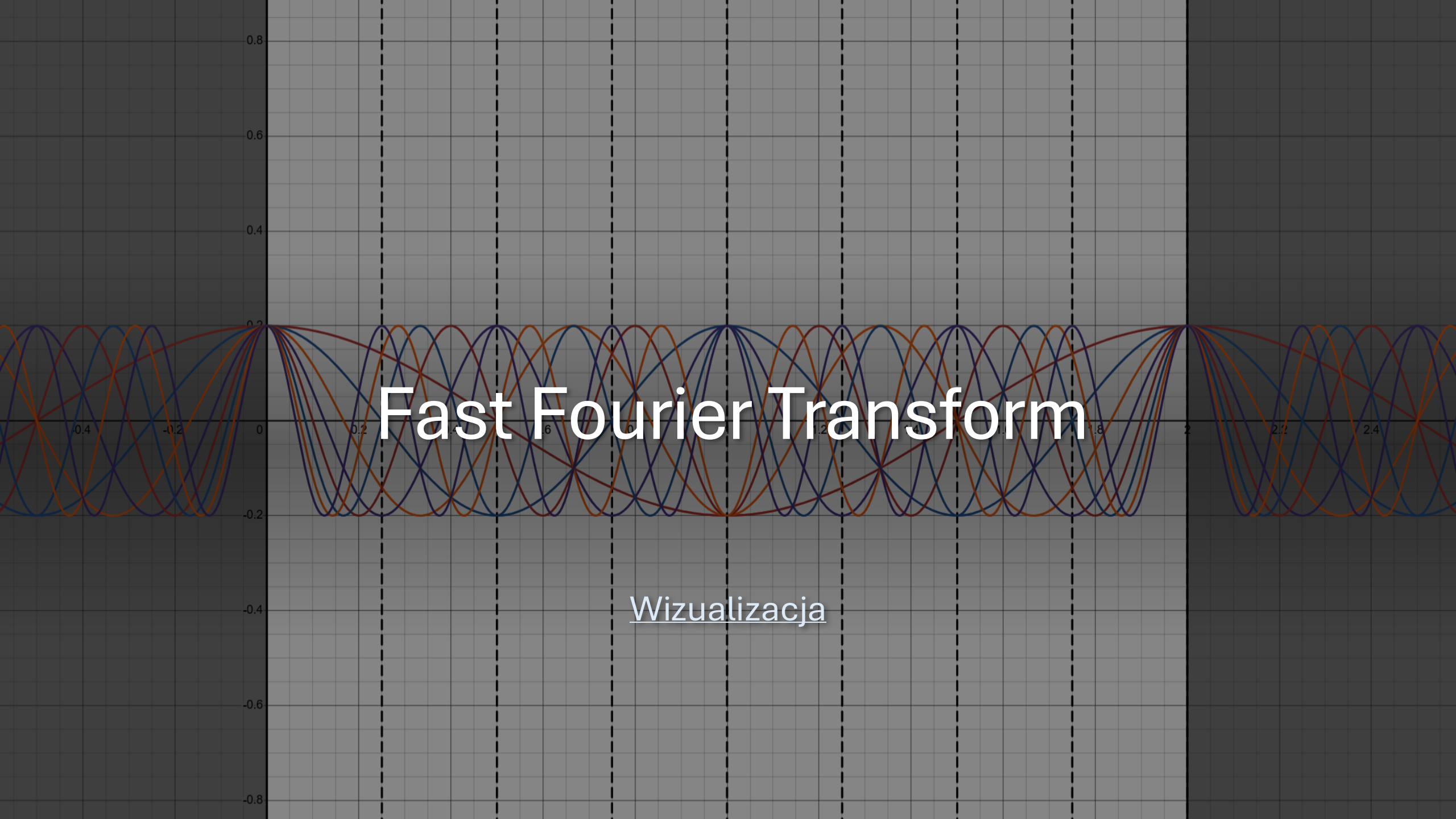
$$A_k = \sum_{n=0}^{N-1} a_n w_N^{-kn}, \ 0 \leqslant k \leqslant N-1$$

$$w_N = e^{i\frac{2\pi}{N}}$$

Richard Garwin

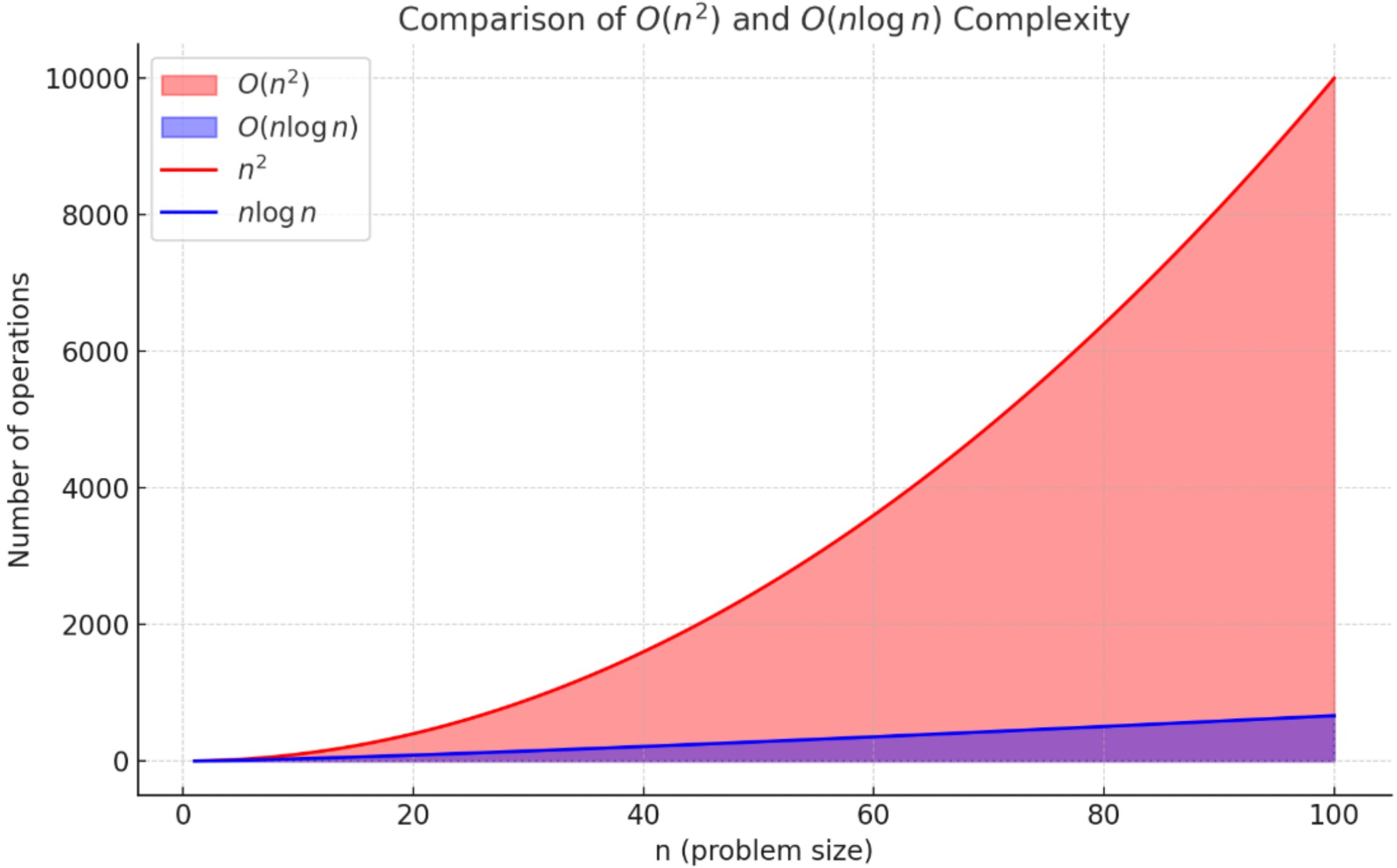
John Tuckey





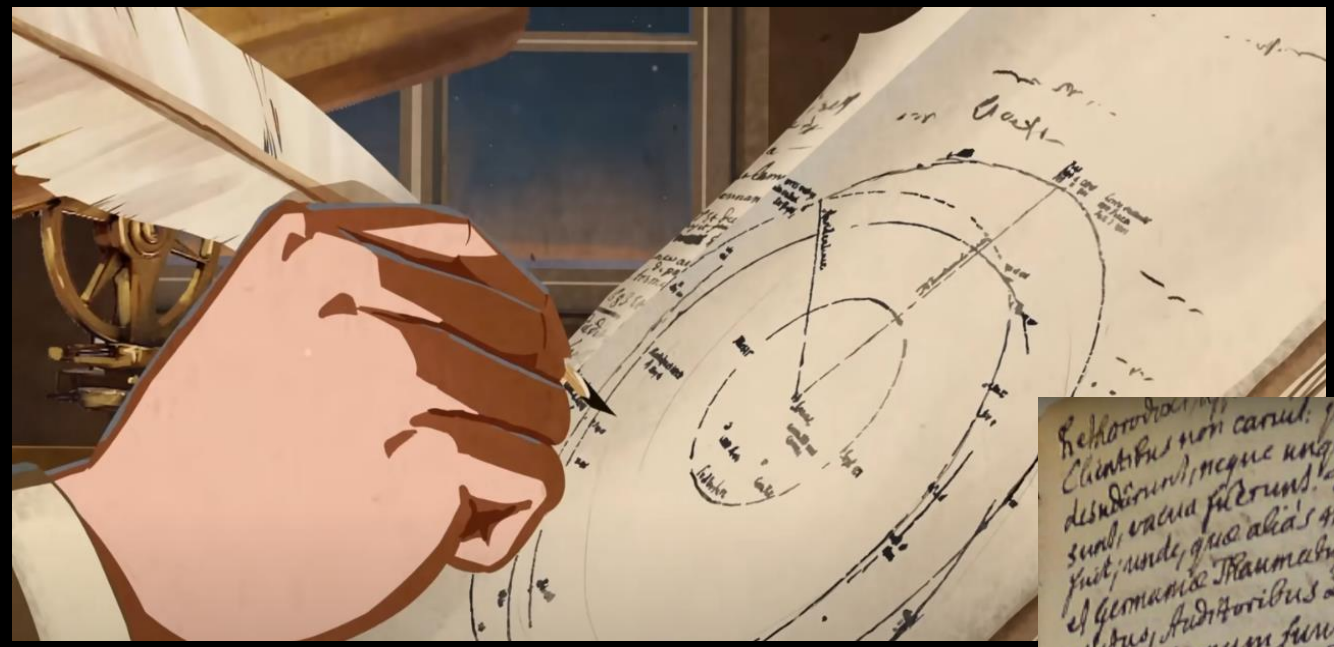
Fast Fourier Transform

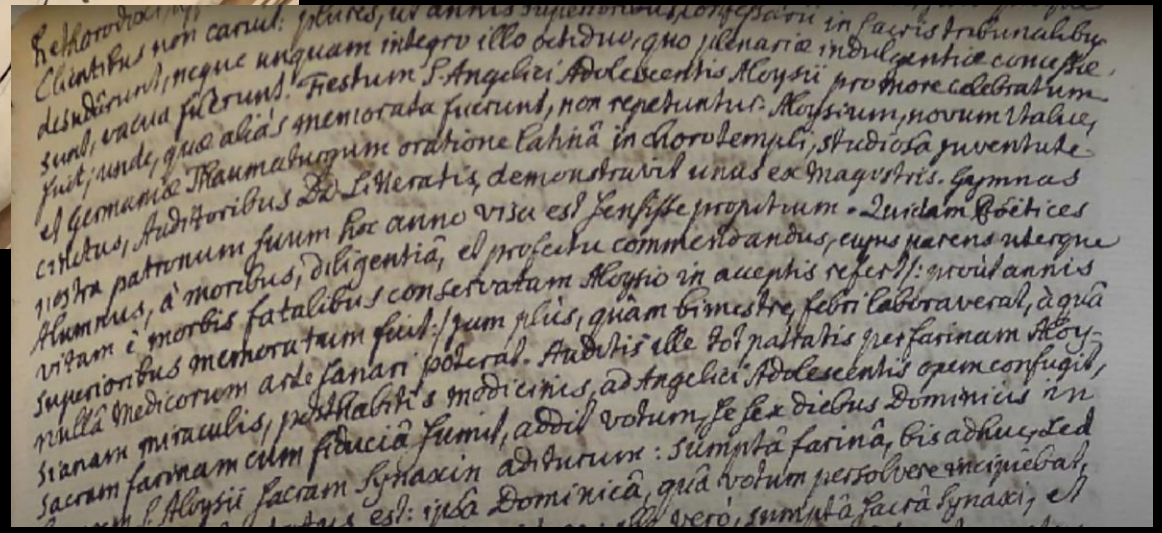
Wizualizacja


Comparison of $O(n^2)$ and $O(n\log n)$ Complexity
$O(n^2)$
$O(n\log n)$
n^2
$n\log n$
Number of operations
10000
8000
6000
4000
2000
0
0
20
40
60
80
100
n (problem size)

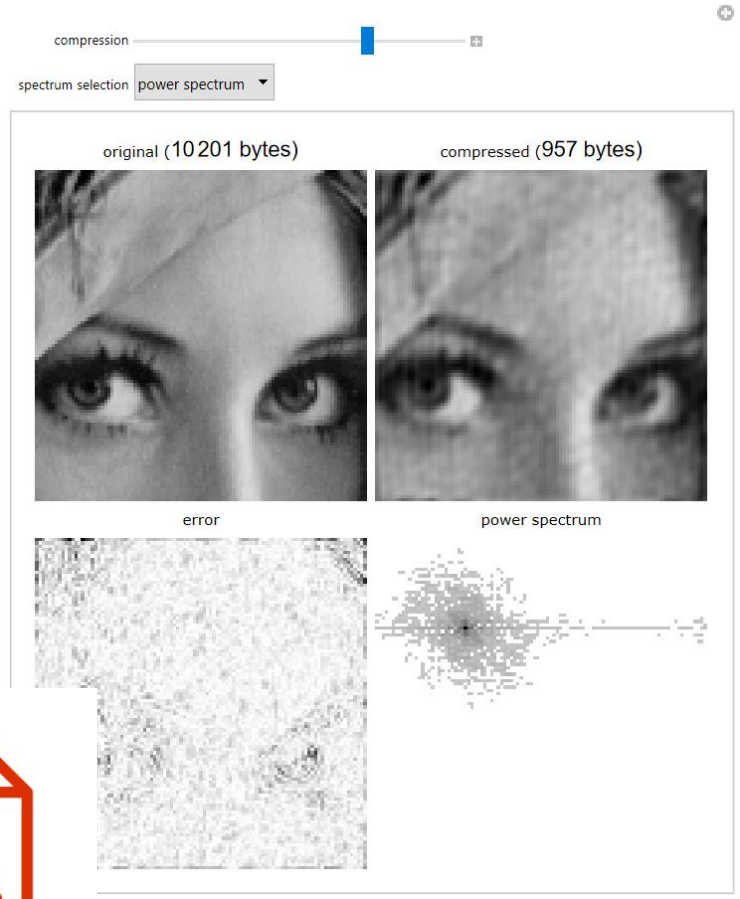
Gauss był pierwszy?

Gauss był pierwszy?

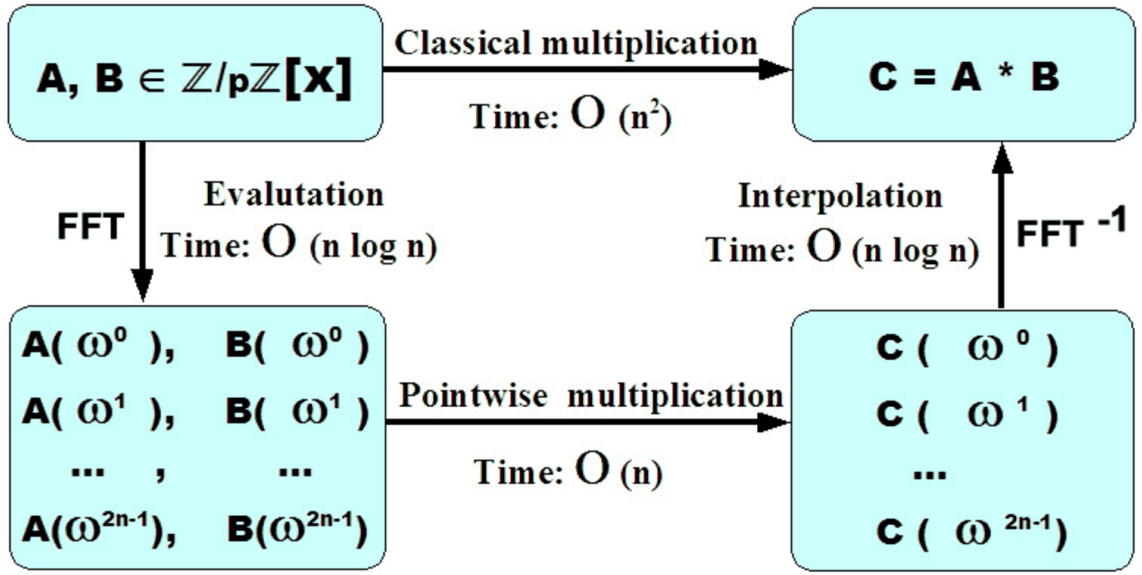




Zastosowania FFT



JPEG



Dziękujemy za uwagę:)

Źródła

- https://youtu.be/nmgFG7PUHfo?si=rwIjN95KNdGVy6-g
- https://pl.wikipedia.org/wiki/Szereg_Fouriera
- https://dibsmethodsmeetings.github.io/fourier-transforms/
- https://pl.wikipedia.org/wiki/Dyskretna_transformata_Fouriera
- https://en.wikipedia.org/wiki/Nevada_Test_Site