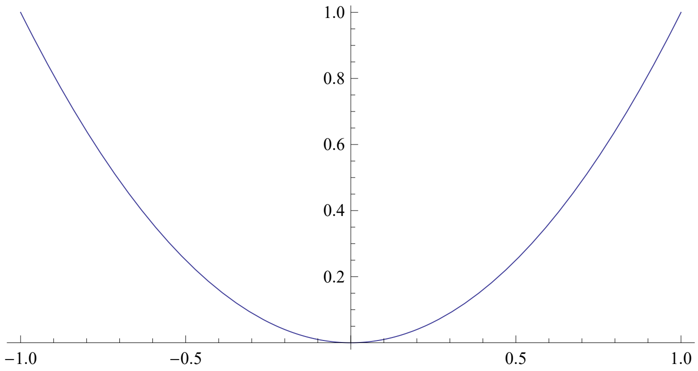
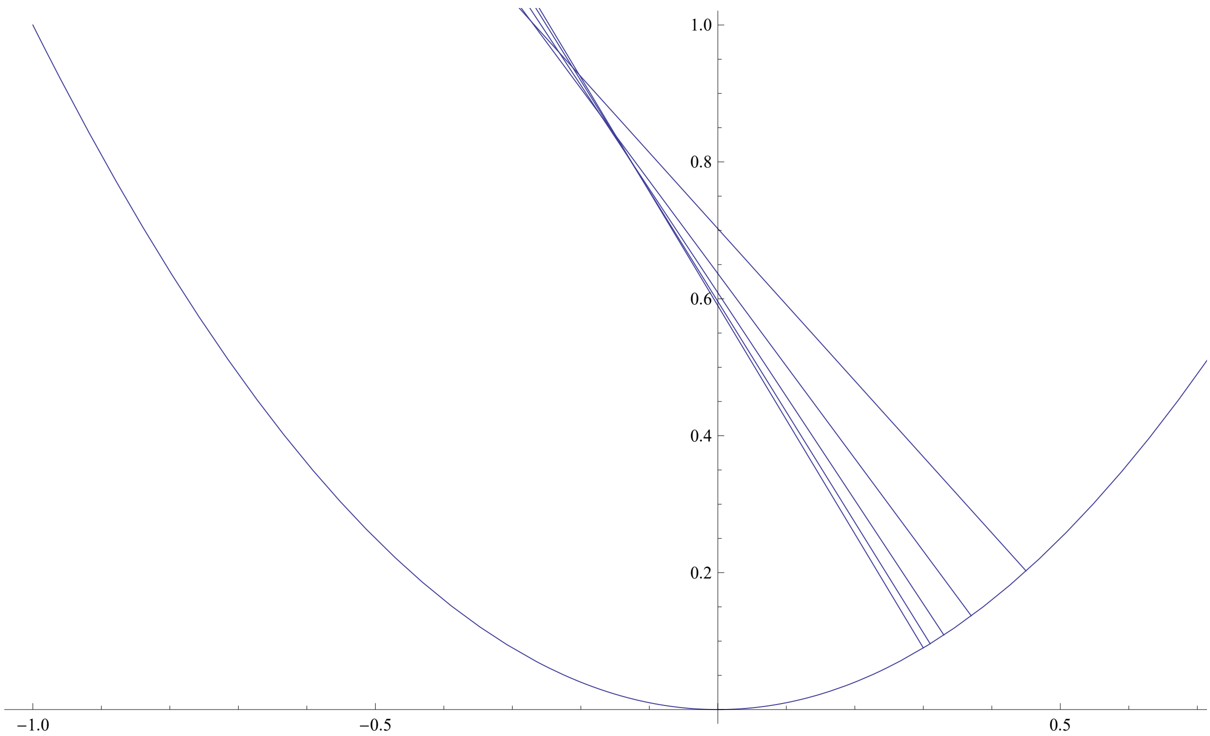
In[60]:=
```
f[t_] = t^2
```
Out[60]= t^2

In[61]:=
```
curve = Plot[f[t], {t, -1, 1}, AspectRatio → Automatic]
```
Out[61]=



In[62]:=
```
df[t_] = D[f[t], t]
```
Out[62]= 2 t

In[63]:=
```
NormalLine[t_] := ParametricPlot[{t - df[t] * s, f[t] + s}, {s, 0, 2}]
```
In[64]:=
```
Show[curve, NormalLine[0.45], NormalLine[0.37],
 NormalLine[0.33], NormalLine[0.31], NormalLine[0.3]]
```
Out[64]=



In[65]:=
```
g[t_] = {g1[t], g2[t]}
```
Out[65]= {g1[t], g2[t]}

In[66]:=
```
dg[t_] = D[g[t], t]
```
Out[66]= {g1′[t], g2′[t]}

In[67]:=

In[68]:=
```
roz = Solve[Dot[dg[t], {x, y} - g[t]] == 0 && Dot[dg[s], {x, y} - g[s]] == 0, {x, y}]
```

Out[68]=
```
{{x → -(-g1[t] g1′[t] g2′[s] + g1[s] g1′[s] g2′[t] +
        g2[s] g2′[s] g2′[t] - g2[t] g2′[s] g2′[t]) / (g1′[t] g2′[s] - g1′[s] g2′[t]),
  y → -(g1[s] g1′[s] g1′[t] - g1[t] g1′[s] g1′[t] + g2[s] g1′[t] g2′[s] -
        g2[t] g1′[s] g2′[t]) / (-g1′[t] g2′[s] + g1′[s] g2′[t])}}
```

In[69]:=
```
G1[s_, t_] = roz[[1, 1, 2]]
```

Out[69]=
```
-(-g1[t] g1′[t] g2′[s] + g1[s] g1′[s] g2′[t] + g2[s] g2′[s] g2′[t] - g2[t] g2′[s] g2′[t]) /
  (g1′[t] g2′[s] - g1′[s] g2′[t])
```

In[70]:=
```
G1[s, s]
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

Out[70]=
```
Indeterminate
```

In[71]:=
```
FullSimplify[-g1[t] g1′[t] g2′[s] + g1[s] g1′[s] g2′[t] +
   g2[s] g2′[s] g2′[t] - g2[t] g2′[s] g2′[t] /. {t → s}]
```

Out[71]= 0

In[72]:=
```
FullSimplify[g1′[t] g2′[s] - g1′[s] g2′[t] /. {t → s}]
```

Out[72]= 0

In[73]:=
```
G2[s_, t_] = roz[[1, 2, 2]]
```

Out[73]=
```
-(g1[s] g1′[s] g1′[t] - g1[t] g1′[s] g1′[t] + g2[s] g1′[t] g2′[s] - g2[t] g1′[s] g2′[t]) /
  (-g1′[t] g2′[s] + g1′[s] g2′[t])
```

In[74]:=
```
G2[s, s]
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

Out[74]=
```
Indeterminate
```

In[75]:=
```
FullSimplify[g1[s] g1′[s] g1′[t] - g1[t] g1′[s] g1′[t] +
   g2[s] g1′[t] g2′[s] - g2[t] g1′[s] g2′[t] /. {t → s}]
```

Out[75]= 0

In[76]:=
```
FullSimplify[-g1′[t] g2′[s] + g1′[s] g2′[t] /. {t → s}]
```

Out[76]= 0

In[77]:=
```
H1[s_, t_] = -D[-g1[t] g1′[t] g2′[s] + g1[s] g1′[s] g2′[t] + g2[s] g2′[s] g2′[t] -
      g2[t] g2′[s] g2′[t], t] / D[g1′[t] g2′[s] - g1′[s] g2′[t], t]
```

Out[77]=
```
-(-g1′[t]^2 g2′[s] - g2′[s] g2′[t]^2 - g1[t] g2′[s] g1″[t] + g1[s] g1′[s] g2″[t] +
     g2[s] g2′[s] g2″[t] - g2[t] g2′[s] g2″[t]) / (g2′[s] g1″[t] - g1′[s] g2″[t])
```

In[78]:= **xCenterOfCurvature[s_] = FullSimplify[H1[s, s]]**

Out[78]= $g1[s] + \frac{g2'[s] \left(g1'[s]^2 + g2'[s]^2\right)}{g2'[s]\, g1''[s] - g1'[s]\, g2''[s]}$

In[79]:=

```
H2[s_, t_] =
  -D[g1[s] g1′[s] g1′[t] - g1[t] g1′[s] g1′[t] + g2[s] g1′[t] g2′[s] - g2[t] g1′[s] g2′[t],
     t] / D[-g1′[t] g2′[s] + g1′[s] g2′[t], t]
```

Out[79]= $-\left(-g1'[s]\, g1'[t]^2 - g1'[s]\, g2'[t]^2 + g1[s]\, g1'[s]\, g1''[t] - g1[t]\, g1'[s]\, g1''[t] + g2[s]\, g2'[s]\, g1''[t] - g2[t]\, g1'[s]\, g2''[t]\right) / \left(-g2'[s]\, g1''[t] + g1'[s]\, g2''[t]\right)$

In[80]:= **yCenterOfCurvature[s_] = FullSimplify[H2[s, s]]**

Out[80]= $g2[s] + \frac{g1'[s] \left(g1'[s]^2 + g2'[s]^2\right)}{-g2'[s]\, g1''[s] + g1'[s]\, g2''[s]}$

In[81]:=

```
RadiusOfCurvatureSquare[s_] =
  FullSimplify[Dot[g[s] - {xx[s], yy[s]}, g[s] - {xx[s], yy[s]}]]
```

Out[81]= $\frac{\left(g1'[s]^2 + g2'[s]^2\right)^3}{\left(g2'[s]\, g1''[s] - g1'[s]\, g2''[s]\right)^2}$

In[82]:= **CurvatureSquare[s_] = 1 / RadiusOfCurvatureSquare[s]**

Out[82]= $\frac{\left(g2'[s]\, g1''[s] - g1'[s]\, g2''[s]\right)^2}{\left(g1'[s]^2 + g2'[s]^2\right)^3}$