

$$\text{In[1]:= } \mathbf{d2pdu2[u_, v_]} = \mathbf{G111[u, v] * dpdu[u, v] + G112[u, v] * dpdv[u, v] + l[u, v] * vi[u, v]}$$

$$\text{Out[1]= } dpdu[u, v] \times G111[u, v] + dpdv[u, v] \times G112[u, v] + l[u, v] \times vi[u, v]$$

$$\text{In[2]:= } \mathbf{d2pdv2[u_, v_]} = \mathbf{G221[u, v] * dpdu[u, v] + G222[u, v] * dpdv[u, v] + n[u, v] * vi[u, v]}$$

$$\text{Out[2]= } dpdu[u, v] \times G221[u, v] + dpdv[u, v] \times G222[u, v] + n[u, v] \times vi[u, v]$$

$$\text{In[3]:= } \mathbf{d2pdu dv[u_, v_]} = \mathbf{G121[u, v] * dpdu[u, v] + G122[u, v] * dpdv[u, v] + m[u, v] * vi[u, v]}$$

$$\text{Out[3]= } dpdu[u, v] \times G121[u, v] + dpdv[u, v] \times G122[u, v] + m[u, v] \times vi[u, v]$$

$$\text{In[4]:= } \mathbf{dvidu[u_, v_]} = \mathbf{a11[u, v] * dpdu[u, v] + a21[u, v] * dpdv[u, v]}$$

$$\text{Out[4]= } a11[u, v] \times dpdu[u, v] + a21[u, v] \times dpdv[u, v]$$

$$\text{In[5]:= } \mathbf{dvidv[u_, v_]} = \mathbf{a12[u, v] * dpdu[u, v] + a22[u, v] * dpdv[u, v]}$$

$$\text{Out[5]= } a12[u, v] \times dpdu[u, v] + a22[u, v] \times dpdv[u, v]$$

$$\text{In[6]:= } \mathbf{detIFF[u_, v_]} = \mathbf{e[u, v] * g[u, v] - f[u, v]^2}$$

$$\text{Out[6]= } -f[u, v]^2 + e[u, v] \times g[u, v]$$

$$\text{In[7]:= } \mathbf{a11[u_, v_]} = \mathbf{(m[u, v] * f[u, v] - l[u, v] * g[u, v]) / detIFF[u, v]}$$

$$\text{Out[7]= } \frac{-g[u, v] \times l[u, v] + f[u, v] \times m[u, v]}{-f[u, v]^2 + e[u, v] \times g[u, v]}$$

$$\text{In[8]:= } \mathbf{a12[u_, v_]} = \mathbf{(n[u, v] * f[u, v] - m[u, v] * g[u, v]) / detIFF[u, v]}$$

$$\text{Out[8]= } \frac{-g[u, v] \times m[u, v] + f[u, v] \times n[u, v]}{-f[u, v]^2 + e[u, v] \times g[u, v]}$$

$$\text{In[9]:= } \mathbf{a21[u_, v_]} = \mathbf{(l[u, v] * f[u, v] - m[u, v] * e[u, v]) / detIFF[u, v]}$$

$$\text{Out[9]= } \frac{f[u, v] \times l[u, v] - e[u, v] \times m[u, v]}{-f[u, v]^2 + e[u, v] \times g[u, v]}$$

$$\text{In[10]:= } \mathbf{a22[u_, v_]} = \mathbf{(m[u, v] * f[u, v] - n[u, v] * e[u, v]) / detIFF[u, v]}$$

$$\text{Out[10]= } \frac{f[u, v] \times m[u, v] - e[u, v] \times n[u, v]}{-f[u, v]^2 + e[u, v] \times g[u, v]}$$

In[11]=

podstaw = {D[dpdu[u, v], u] → d2pdu2[u, v], D[dpdu[u, v], v] → d2pdudv[u, v],
 D[dpdv[u, v], u] → d2pdudv[u, v], D[dpdv[u, v], v] → d2pdv2[u, v],
 D[vi[u, v], u] → dvidu[u, v], D[vi[u, v], v] → dvidv[u, v]}

Out[11]= { dpdu^(1,0)[u, v] → dpdu[u, v] × G111[u, v] + dpdv[u, v] × G112[u, v] + l[u, v] × vi[u, v],
 dpdu^(0,1)[u, v] → dpdu[u, v] × G121[u, v] + dpdv[u, v] × G122[u, v] + m[u, v] × vi[u, v],
 dpdv^(1,0)[u, v] → dpdu[u, v] × G121[u, v] + dpdv[u, v] × G122[u, v] + m[u, v] × vi[u, v],
 dpdv^(0,1)[u, v] → dpdu[u, v] × G221[u, v] + dpdv[u, v] × G222[u, v] + n[u, v] × vi[u, v],
 vi^(1,0)[u, v] → $\frac{dpdv[u, v] (f[u, v] \times l[u, v] - e[u, v] \times m[u, v])}{-f[u, v]^2 + e[u, v] \times g[u, v]} +$
 $\frac{dpdu[u, v] (-g[u, v] \times l[u, v] + f[u, v] \times m[u, v])}{-f[u, v]^2 + e[u, v] \times g[u, v]},$
 vi^(0,1)[u, v] → $\frac{dpdv[u, v] (f[u, v] \times m[u, v] - e[u, v] \times n[u, v])}{-f[u, v]^2 + e[u, v] \times g[u, v]} +$
 $\frac{dpdu[u, v] (-g[u, v] \times m[u, v] + f[u, v] \times n[u, v])}{-f[u, v]^2 + e[u, v] \times g[u, v]} \}$

In[13]= Schpuuv[u_, v_] = FullSimplify[(D[d2pdu2[u, v], v] - D[d2pdudv[u, v], u]) /. podstaw]
 Łuprość pełniej Łoblicz pochodną Łoblicz pochodną

Out[13]= $\frac{1}{f[u, v]^2 - e[u, v] \times g[u, v]}$
 (dpdu[u, v] (f[u, v] (m[u, v]² - l[u, v] × n[u, v]) + f[u, v]² (-G121[u, v] × G122[u, v] +
 G112[u, v] × G221[u, v] + G111^(0,1)[u, v] - G121^(1,0)[u, v]) +
 e[u, v] × g[u, v] (G121[u, v] × G122[u, v] - G112[u, v] × G221[u, v] -
 G111^(0,1)[u, v] + G121^(1,0)[u, v])) +
 dpdv[u, v] (f[u, v]² ((G111[u, v] - G122[u, v]) G122[u, v] +
 G112[u, v] (-G121[u, v] + G222[u, v]) + G112^(0,1)[u, v] - G122^(1,0)[u, v]) +
 e[u, v] (-m[u, v]² + l[u, v] × n[u, v] + g[u, v] (-G111[u, v] × G122[u, v] + G122[u, v]² +
 G112[u, v] (G121[u, v] - G222[u, v]) - G112^(0,1)[u, v] + G122^(1,0)[u, v])) +
 (f[u, v]² - e[u, v] × g[u, v]) vi[u, v] (-G121[u, v] × l[u, v] +
 (G111[u, v] - G122[u, v]) m[u, v] + G112[u, v] × n[u, v] + l^(0,1)[u, v] - m^(1,0)[u, v]))

In[14]= puuvdpdv[u_, v_] = FullSimplify[Coefficient[Schpuuv[u, v], dpdv[u, v]]]
 Łuprość pełniej Łwspółczynnik

Out[14]= $\frac{1}{f[u, v]^2 - e[u, v] \times g[u, v]}$
 (f[u, v]² ((G111[u, v] - G122[u, v]) G122[u, v] + G112[u, v] (-G121[u, v] + G222[u, v]) +
 G112^(0,1)[u, v] - G122^(1,0)[u, v]) +
 e[u, v] (-m[u, v]² + l[u, v] × n[u, v] + g[u, v] (-G111[u, v] × G122[u, v] + G122[u, v]² +
 G112[u, v] (G121[u, v] - G222[u, v]) - G112^(0,1)[u, v] + G122^(1,0)[u, v]))

In[15]= **numpuuvdpdv**[u_, v_] = **FullSimplify**[**Numerator**[puuvdpdv[u, v]]]
[uprość pełniej] [licznik]

Out[15]= $f[u, v]^2 \left((G111[u, v] - G122[u, v]) G122[u, v] + \right.$
 $G112[u, v] (-G121[u, v] + G222[u, v]) + G112^{(0,1)}[u, v] - G122^{(1,0)}[u, v] \left. \right) +$
 $e[u, v] \left(-m[u, v]^2 + l[u, v] \times n[u, v] + g[u, v] \left(-G111[u, v] \times G122[u, v] + G122[u, v]^2 + \right. \right.$
 $G112[u, v] (G121[u, v] - G222[u, v]) - G112^{(0,1)}[u, v] + G122^{(1,0)}[u, v] \left. \right) \left. \right)$

In[16]=

In[17]= **FullSimplify**[**Coefficient**[numpuuvdpdv[u, v], f[u, v]^2] +
[uprość pełniej] [współczynnik]
Coefficient[numpuuvdpdv[u, v], e[u, v] * g[u, v]]]
[współczynnik]

Out[17]= 0

In[18]= **row**[u_, v_] = **Coefficient**[numpuuvdpdv[u, v], f[u, v]^2] (f[u, v]^2 - e[u, v] * g[u, v]) +
[współczynnik]
e[u, v] (-m[u, v]^2 + l[u, v] * n[u, v])

Out[18]= $e[u, v] \left(-m[u, v]^2 + l[u, v] \times n[u, v] \right) +$
 $\left(f[u, v]^2 - e[u, v] \times g[u, v] \right) \left((G111[u, v] - G122[u, v]) G122[u, v] + \right.$
 $G112[u, v] (-G121[u, v] + G222[u, v]) + G112^{(0,1)}[u, v] - G122^{(1,0)}[u, v] \left. \right)$

In[19]= **FullSimplify**[**row**[u, v] - numpuuvdpdv[u, v]]
[uprość pełniej]

Out[19]= 0

In[20]= **frow**[u_, v_] = **FullSimplify**[**row**[u, v] / **Denominator**[puuvdpdv[u, v]]] /.
[uprość pełniej] [mianownik]

$$\left\{ \frac{(m[u, v]^2 - l[u, v] \times n[u, v])}{-f[u, v]^2 + e[u, v] \times g[u, v]} \rightarrow -K[u, v] \right\}$$

Out[20]= $(G111[u, v] - G122[u, v]) G122[u, v] + G112[u, v] (-G121[u, v] + G222[u, v]) -$
 $e[u, v] \times K[u, v] + G112^{(0,1)}[u, v] - G122^{(1,0)}[u, v]$

In[21]= **FullSimplify**[**Solve**[frow[u, v] == 0, K[u, v]]]
[uprość pełniej] [rozwiąż równanie]

Out[21]= $\left\{ \left\{ K[u, v] \rightarrow \frac{1}{e[u, v]} \left((G111[u, v] - G122[u, v]) G122[u, v] + \right. \right. \right.$
 $G112[u, v] (-G121[u, v] + G222[u, v]) + G112^{(0,1)}[u, v] - G122^{(1,0)}[u, v] \left. \left. \right) \right\} \right\}$