RISK THEORY AND INSURANCE APPLICATIONS

Tutorial/lab, March 15 and 22, 2010

**Exc. 1.** The policyholders of an automobile insurance company fall into two classes. In the class *k, k* = 1, 2, there are *nk* policies, with claim probability *qk* and with the probability distribution function of claim amount *Bk*, in the case of an accident, of the form

 Let *n*1 = 500, *n*2 = 2000, *q*1 = 0.1, *q*2 = 0.05, *λ*1 = 1, *λ*2 = 2, *L*1 = 2.5, *L*2 = 5. The company wants to collect from this portfolio an amount (premium) equal to 95-th percentile (that is the quantile of order 0.95) of the distribution of aggregate claims *S*. Moreover it wants each policyholder’s share this amount to be proportional to that individual’s expected claim. The share for policyholder *j* with mean loss would be This extra amount: is called the security loading and *θ* is the relative security loading. Calculate *θ*. (Hint: use the normal approximation for the distribution of *S*).

**Exc. 2**. There are two independent risks *X* and *Y* with probability density functions *g* and *h*, respectively, where for any *x* > 0 we have:

, .

Find the probability density function of *S* = *X* + *Y*. (Hint: use 2 methods

* direct calculation of the convolution:

,

* identify the distribution of *S* based on the moment generating function of *S.*

**Exc. 3**. Consider an insurance portfolio that will produce 0, 1, 2, or 3 claims in a fixed time period with probabilities 0.1, 0.3, 0.4, and 0.2, respectively. An individual claim will be of amount 1, 2, or 3 with probabilities 0.5, 0.4 and 0.1, respectively. Calculate the probability function of the aggregate claims.

**Exc. 4.** Let be *k* different numbers and suppose that be independent random variables such that, for has a Poisson distribution with parameter . What is the distribution of ?

**Exc. 5.** Suppose that the aggregate claims *S* generated by the portfolio for the period under study has a compound Poisson distribution with *λ* = 0.8 and individual claims amounts that are 1, 2 or 3 with probabilities 0.25, 0.375 and 0.375, respectively. Calculate values of the probability function of *S* for *x* = 0, 1, 2, …, 6. (Hint: there are 3 methods of calculations – direct method, method using properties of the compound Poisson distribution, and the Panjer recursion algorithm, use each of them).

**Exc**. **6**. Suppose that the number of accidents generated by an insured driver in a single year has a Poisson distribution with parameter *λ*. If an accident happens the probability is *p* that the damage amount will exceed a deductible amount. On the assumption that the number of accidents is independent of the severity of the accidents, derive the distribution of the number of accidents that result in a claim payment.

**Exc. 7.** Suppose that *S*1 has a compound Poisson distribution with Poisson parameter *λ* = 2 and claim amounts that are 1, 2 of 3 with probabilities 0.2, 0.6 and 0.2 respectively. Additionally *S*2 has a compound Poisson distribution with Poisson parameter *λ* = 6 and claim amounts that are either 3 or 4 with probability 0.5 each. Suppose that *S*1, *S*2 are independent risks. What is the distribution of ?

**Exc. 8**. Suppose that *S* has a compound Poisson distribution with *λ* = 2 and Calculate values of the probability function of *S* for *x* = 0, 1, 2,3,4.