Automata Theory and Formal Languages
Class 8

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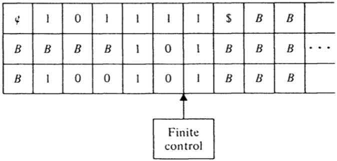
Multitrack Turing Machine



Rysunek 1: Multitrack Turing Machine

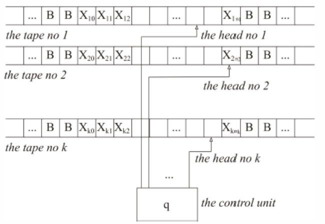
- Multitrack Turing Machine is a modification of TM in which the head read k symbols from k tracks at once.
- It differs from MT with the transition function

$$\delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R\}$$

- One can see that multitrack TM is an equivalent of TM with symbols defined as the projection of alphabet Γ^k symbols.

Multitape Turing Machine

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$



Rysunek 2: Multitape Turing Machine

$\Gamma_1, \Gamma_2, \ldots, \Gamma_k$ are tape alphabets

δ the transition function

$$Q \times (\Gamma_1 \times \Gamma_2 \times \cdots \times \Gamma_k) \rightarrow Q \times (\Gamma_1 \times \Gamma_2 \times \cdots \times \Gamma_k) \times \{L, R, S\}^k$$

Machines equivalence

1. A one tape TM is a special case of Multitape TM ($k = 1$).
2. Multitape TM can be simulated using Multitrack TM.

Simulator architecture

Head 1		X				
Tape 1	A_1	A_2	$\cdots$		$\cdots$	A_m
Head 2				X		
Tape 2	B_1	B_2	$\cdots$		$\cdots$	B_m
Head 3	X					
Tape 3	C_1	C_2	$\cdots$		$\cdots$	C_m

Rysunek 3: Simulation

- Each from k tapes of simulated TM M is simulated using two tracks.
  1. A lower track contains content of i-th tape of TM M.
  2. An upper track contains only one symbol X meaning a position of the head over i-th tape .
- In such way, we create machine N with $2 * k$ tracks.

Simulations

- To simulate a transition of M the machine N must visit all k head markers.
- For each marker – according to the state M and a symbol pointed by the marker – we change the symbol.
- If it is necessary, we move the marker one cell left or right.
- When all changes on tracks are done, we change the state of TM N into the state adequate for TM M transition. If the state is the final one, we accept the calculations.

Computing time

Simulation time

The time necessary for MT N to simulate n transitions k-tape TM M is $O(n^2)$.

- After n simulated transitions, the distance between furthest head markers cannot be higher than $2n$ cells.
- Starting from the first marker, TM N needs no more than $2n$ movements to reach the last one.
- Next, TM needs no more than $2n$ transitions to get back to the first marker. During the movements, the machine simulates changes on the tapes.
- No more than $2k$ movements to modify positions of the markers.
- This gives $4n + 2k \sim O(n)$ movements for a single movement simulation and $O(n^2)$ to simulate n movements.

Task

- Design multitape MT to calculate $f(n) = 2^n$.

T¡ architecture

- We use two-way infinite TM.
- We use three tapes:
  1. Input tape, iteration counter.
  2. The result of the last iteration.
  3. Doubled result

Algorithm draft

1. The first tape defines the number of iterations $c = n$.
2. The second tape contains $o = 1$.
3. In each iteration $1 \dots n$:
   3.1 On the first tape $c = c - 1$
   3.2 On the third tape $o = 2o$
   3.3 We rewrite the third tape on the second one.
4. We set o as the result on the first tape.

Transition table

δ	$\begin{pmatrix}1\\B\\B\end{pmatrix}$	$\begin{pmatrix}B\\B\\B\end{pmatrix}$	$\begin{pmatrix}1\\1\\B\end{pmatrix}$	$\begin{pmatrix}*\\B\\1\end{pmatrix}$
q_0	$\left(q_1, \begin{pmatrix}1\\1\\B\end{pmatrix}, \begin{pmatrix}S\\S\\S\end{pmatrix}\right)$	$\left(q_A, \begin{pmatrix}1\\B\\B\end{pmatrix}, \begin{pmatrix}S\\S\\S\end{pmatrix}\right)$	–	–
q_1	$\left(q_3, \begin{pmatrix}B\\B\\B\end{pmatrix}, \begin{pmatrix}R\\S\\L\end{pmatrix}\right)$	–	$\left(q_2, \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}S\\S\\R\end{pmatrix}\right)$	–
q_2	–	–	$\left(q_1, \begin{pmatrix}1\\B\\1\end{pmatrix}, \begin{pmatrix}S\\R\\R\end{pmatrix}\right)$	–
q_3	$\left(q_1, \begin{pmatrix}1\\B\\B\end{pmatrix}, \begin{pmatrix}S\\R\\S\end{pmatrix}\right)$	$\left(q_4, \begin{pmatrix}B\\B\\B\end{pmatrix}, \begin{pmatrix}S\\L\\S\end{pmatrix}\right)$	–	$\left(q_3, \begin{pmatrix}*\\1\\B\end{pmatrix}, \begin{pmatrix}S\\R\\L\end{pmatrix}\right)$

δ	$\begin{pmatrix}B\\1\\B\end{pmatrix}$	$\begin{pmatrix}B\\B\\B\end{pmatrix}$
q_4	$\left(q_4, \begin{pmatrix}1\\B\\B\end{pmatrix}, \begin{pmatrix}L\\L\\S\end{pmatrix}\right)$	$\left(q_A, \begin{pmatrix}B\\B\\B\end{pmatrix}, \begin{pmatrix}R\\S\\S\end{pmatrix}\right)$

The final model

$$
\begin{aligned}
M = (Q = & \quad \{q_0, \ldots, q_4, q_A\}, \\
\Sigma = & \quad \{1\}, \\
\Gamma_x = & \quad \{1, B\}, \\
& \quad \delta, \\
& \quad q_0, \\
& \quad B, \\
F = & \quad \{q_A\})
\end{aligned}
$$

Task

Design multitape TM to model language $L = \{a^i b^j c^k : i = j = k\}$ over alphabet $\Sigma = \{a, b, c\}$

TM architecture

- We use TM with two-way infinite tape.
- We use three tapes:
  1. The input tape, sequence c.
  2. Sequence a.
  3. Sequence b.

Transition table

δ	$\begin{pmatrix}a\\B\\B\end{pmatrix}$	$\begin{pmatrix}b\\B\\B\end{pmatrix}$	$\begin{pmatrix}c\\B\\B\end{pmatrix}$	$\begin{pmatrix}B\\B\\B\end{pmatrix}$
q_S	$\left(q_0, \begin{pmatrix}B\\a\\B\end{pmatrix}, \begin{pmatrix}R\\R\\S\end{pmatrix}\right)$	q_R	q_R	q_A
q_0	$\left(q_0, \begin{pmatrix}B\\a\\B\end{pmatrix}, \begin{pmatrix}R\\R\\S\end{pmatrix}\right)$	$\left(q_1, \begin{pmatrix}B\\B\\b\end{pmatrix}, \begin{pmatrix}R\\S\\R\end{pmatrix}\right)$	q_R	q_R
q_1	q_R	$\left(q_1, \begin{pmatrix}B\\B\\b\end{pmatrix}, \begin{pmatrix}R\\S\\R\end{pmatrix}\right)$	$\left(q_2, \begin{pmatrix}c\\B\\B\end{pmatrix}, \begin{pmatrix}S\\S\\S\end{pmatrix}\right)$	q_R
q_2	q_R	q_R	$\left(q_2, \begin{pmatrix}c\\B\\B\end{pmatrix}, \begin{pmatrix}R\\S\\S\end{pmatrix}\right)$	$\left(q_3, \begin{pmatrix}B\\B\\B\end{pmatrix}, \begin{pmatrix}L\\L\\L\end{pmatrix}\right)$

δ	$\begin{pmatrix}c\\B\\b\end{pmatrix}$	$\begin{pmatrix}c\\a\\B\end{pmatrix}$	$\begin{pmatrix}c\\B\\B\end{pmatrix}$	$\begin{pmatrix}B\\a\\b\end{pmatrix}$	$\begin{pmatrix}B\\a\\B\end{pmatrix}$	$\begin{pmatrix}B\\B\\b\end{pmatrix}$	$\begin{pmatrix}B\\B\\B\end{pmatrix}$	$\begin{pmatrix}c\\a\\b\end{pmatrix}$
q_3	q_R	q_R	q_R	q_R	q_R	q_R	q_A	$\left(q_3, \begin{pmatrix}B\\B\\B\end{pmatrix}, \begin{pmatrix}L\\L\\L\end{pmatrix}\right)$

The final model

$$
\begin{aligned}
M = (Q = &\quad \{q_S, q_0, \ldots, q_3, q_A, q_R\},\\
\Sigma = &\quad \{a, b, c\},\\
\Gamma = &\quad \{a, b, c, B\},\\
&\quad \delta,\\
&\quad q_S,\\
&\quad B,\\
F = &\quad \{q_A\},\\
R = &\quad \{q_R\})
\end{aligned}
$$

Assignments I

1. Design a multitape TM to calculate the following functions
   1.1 $f(n, m) = n * m$
   1.2 $f(n_1, \ldots, n_m) = n_1 + n_2 + \cdots + n_m$

Assignments II

2. Design TM to model the following languages
   2.1 Language L over alphabet $\Sigma = \{0, 1\}$ words with the same number of zeros and ones.
   2.2 $L = \{a^i b^j c^k : k = \max(i, j)\}$ over alphabet $\Sigma = \{a, b, c\}$