Automata Theory and Formal Languages
Class 10

Marcin Luckner, PhD
mluckner@mini.pw.edu.pl

Version 1.2
16 stycznia 2022

Nondeterministic Turing machine

A nondeterministic Turing machine differs from the deterministic variety by having a transition function δ mapping to a set of triplets $\{Q \times \Gamma \times \{L, R\}\}$

$$\delta : Q \times \Gamma \rightarrow \{Q \times \Gamma \times \{L, R\}\}^k$$

Nondeterministic calculations

- Nondeterministic calculations differ from the deterministic model when $k > 1$.
- When $k > 1$, a tree structure of calculation is created, each branch determines a selection of different from k transitions.
- From this moment, the calculations are parallel.
- If some calculations in some calculation tree branch finished in the acceptance state, the machine accepts the input.

Guessing process

- Nondeterminism is used to guess that the given symbol that is a common letter from the tape algorithm has a particular position in the word structure.
- For example:
  - it splits a word into half,
  - after this symbols some special symbols do not occur.
- We cannot guess in case of a deterministic machine. We have to check if the symbol has some special position by additional calculations.

Nondeterministic Turing Machine

Design Nondeterministic Turing Machine to accept words in the form of *ww* over alphabe $\Sigma = \{T, F\}$.

Idea

- In this case, we have to guest when the second w starts.
- We use the following algorithm:
  1. Identify the first symbol in w
  2. Find a copy of the symbol
  3. Guest if it is the beginning of the second w
  4. Compare both w

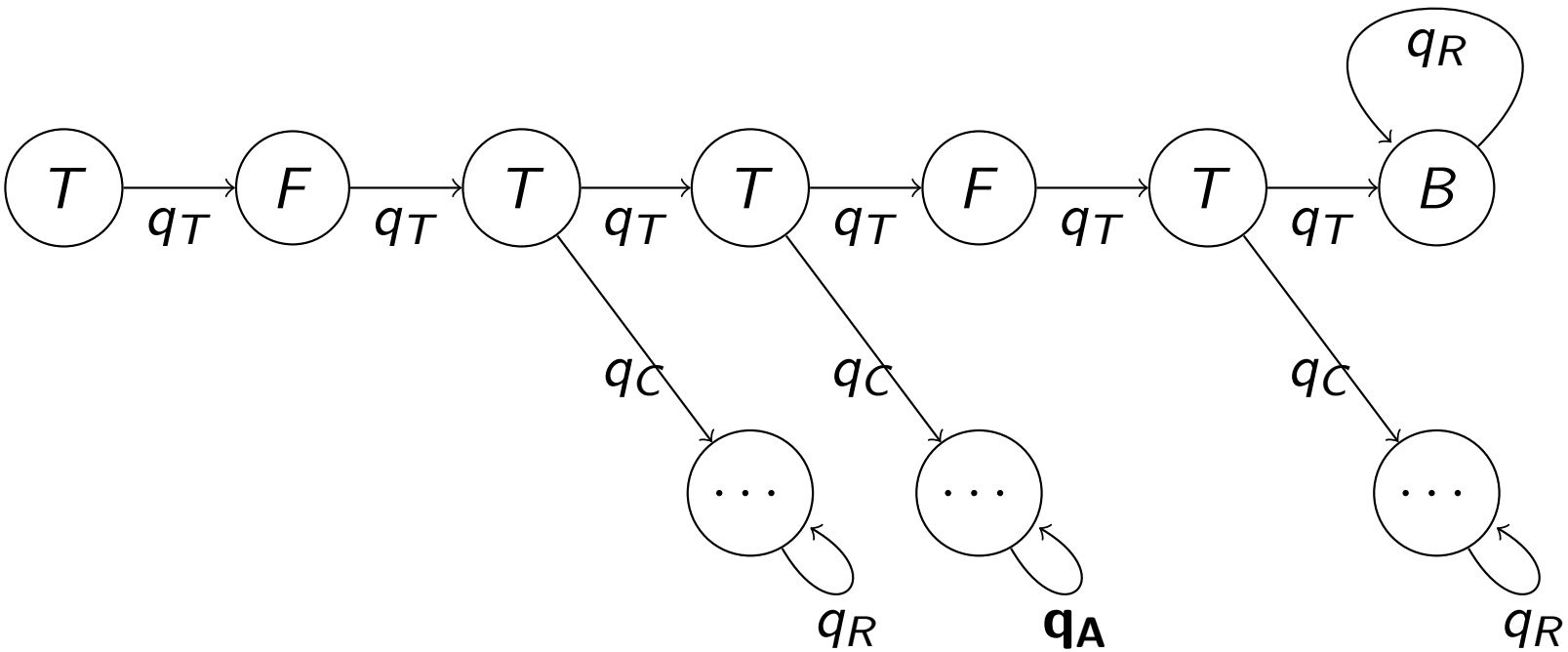
Transition function

δ	T	F	B	X
q_0	(q_T, B, R)	(q_F, B, R)	q_A	$-$
q_T	(q_T, T, R) (q_C, X, R)	(q_T, F, R)	q_R	$-$
q_F	(q_F, T, R)	(q_F, F, R) (q_C, X, R)	q_R	$-$
q_C	(q_1, X, L)	(q_2, X, L)	q_A	(q_C, X, R)
q_1	(q_1, T, L)	(q_1, F, L)	(q_3, B, R)	(q_1, X, L)
q_2	(q_2, T, L)	(q_2, F, L)	(q_4, B, R)	(q_2, X, L)
q_3	(q_5, B, R)	q_R	q_R	q_R
q_4	q_R	(q_5, B, R)	q_R	q_R
q_5	(q_5, T, R)	(q_5, F, R)	$-$	(q_C, X, R)

The final model

$$
\begin{aligned}
M = (Q = & \{q_0, \ldots, q_5, q_A, q_R, q_T, q_F, q_C\}, \\
\Sigma = & \{T, F\}, \\
\Gamma = & \{T, F, X, B\}, \\
& \delta, \\
& q_0, \\
& B, \\
F = & \{q_A\} \\
R = & \{q_R\})
\end{aligned}
$$

Model calculations



Assignments I

1. Design nondeterministic Turing machines to recognise the following languages
   - 1.1 The language of words over the alphabet $\Sigma = \{0, 1\}$ in the form of $w^R w$, where w^R is a reflection of w.
   - 1.2 The language of strings with a length that is not a prime number.
   - 1.3 The list of binary numbers, separated by commas, for which at least one number j is at the jth position.