Automata Theory and Formal Languages
Class 12

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Finite automation models

- All finite automata models consist of the came components

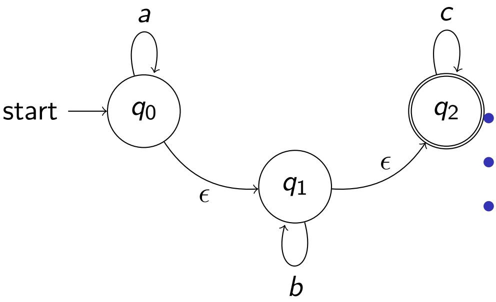
$$M = (Q, \Sigma, \delta, q_0, F)$$

- The only difference is in a transition function
  - automaton with ϵ-movements: $\delta : Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^Q$
  - nondeterministic automaton $\delta : Q \times \Sigma \rightarrow 2^Q$
  - deterministic automaton $\delta : Q \times \Sigma \rightarrow Q$
- One can easy observe that:
  1. Each Nondeterministic Finite Automaton is an automaton with ϵ-movements.
  2. Each deterministic Finite Automaton is a Nondeterministic automaton.
- It can be proved, by construction, that:
  1. Each ϵ-movements automaton can be transferred into NFA.
  2. Each NFA can be transferred into DFA

ϵ–closure

- $\epsilon - Cl(q)$ for state q is a set od all states that can be reached from q with ϵ-movements.

$$\epsilon - Cl(q) = \{p \in Q : p = \delta(q, \epsilon)\} \text{ for } q \in Q$$

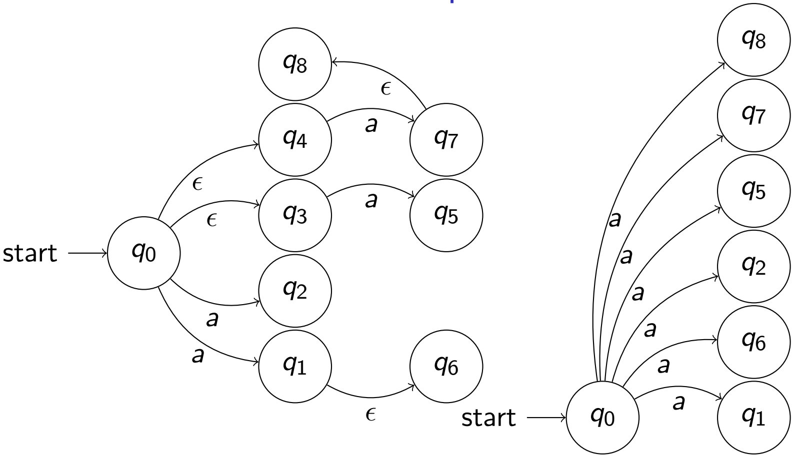


- $\epsilon - Cl(q_0) = \{q_0, q_1, q_2\}$
- $\epsilon - Cl(q_1) = \{q_1, q_2\}$
- $\epsilon - Cl(q_2) = \{q_2\}$

Elimination of ϵ-movements

- To create a structure of nondeterministic equivalent of an automaton with ϵ-movements it is necessary to apply the following algorithm for each state q_i and for each $a \in \Sigma$:
  1. Find $\epsilon - Cl(q_i)$.
  2. Find all a-movements from $\epsilon - Cl(q_i)$. Label the set of obtained states as $Q_a = \{q : \exists(s \in \epsilon - Cl(q_i))\delta(s, a) = q\}$.
  3. For each state $q_a \in Q_a$ calculate $\epsilon - Cl(q_a)$. Label the union as $Q_i = \bigcup_{q_a \in Q_a} \epsilon - Cl(q_a)$.
  4. For each $q_j \in Q_i$ create a transition from q_i into q_j, labelled with a.

Example



$$\epsilon - Cl(q_0) = \{q_0, q_3, q_4\}; \quad (1)$$

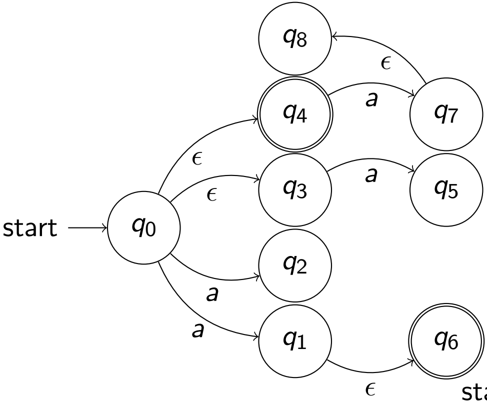
$$Q_a = \{q_1, q_2, q_5, q_7\}; \quad (2)$$

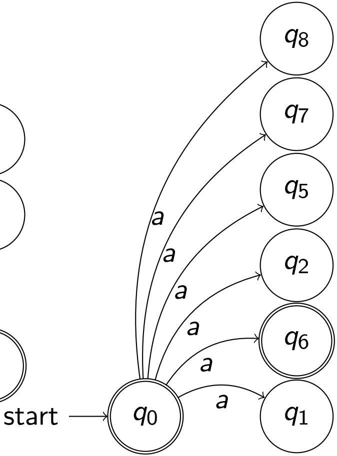
$$Q_i = \{q_1, q_6, q_2, q_5, q_7, q_8\}; \quad (3)$$

Start and final states

- The start state q_0 stay the same in a model with ϵ-movements and its nondeterministic equivalent.
- The final states F stay the same in a model with ϵ-movements and its nondeterministic equivalent.
- However, if $\epsilon - Cl(q_0) \cap F \neq \emptyset$ state q_0 will be a final state in the new model.
    - During elimination of ϵ-movements, we can eliminate acceptance of the empty word ϵ.

Example


start
q0
q1
q2
q3
q4
q5
q6
q7
q8
ϵ
ϵ
a
a
a
a
ϵ
ϵ


start
q0
q1
q6
q2
q5
q7
q8
a
a
a
a
a
a

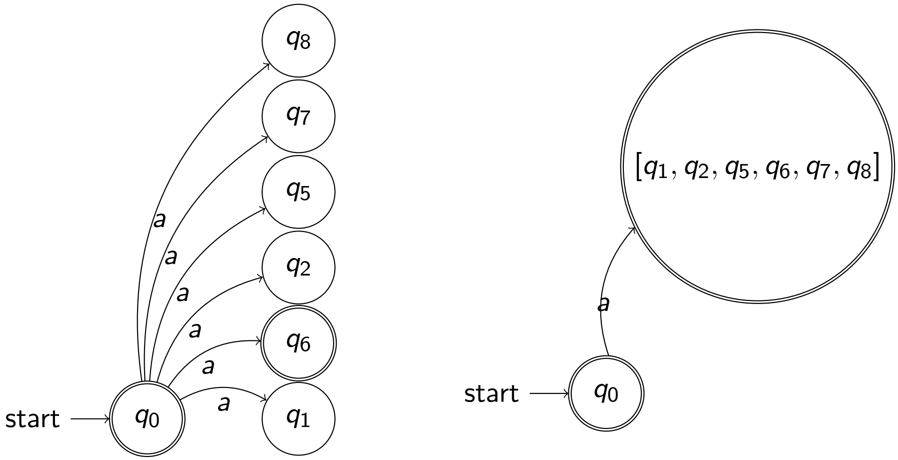
Elimination of nondeterminism

- Nondeterminism can be eliminated by replacing nondeterministic transitions to many states with a deterministic transition to a new meta state.
- The elimination is iterative and starts from the start state q_0.
  1. For q_0 and for each $a \in \Sigma$ Find all a-movements from q_0. Label the set of obtained states as Q_a.
  2. Create a new state $[Q_a]$ labelled with names of all states from Q_a.
  3. Calculate transitions from new created states for each $a \in \Sigma$ remembering that

$$\delta([Q], a) = \left[\bigcup_{q \in Q} \delta(q, a) \right]$$

  4. A new state is a final state if there is a final state in its label.

Example

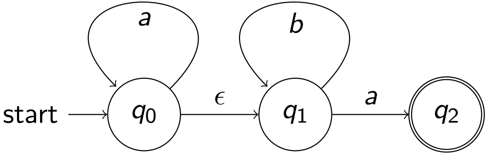


Task

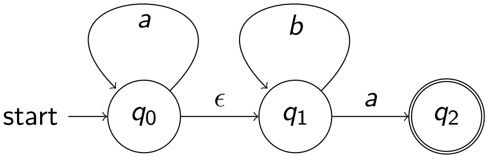
Design deterministic finite automaton equivalent for the following automaton.

	a	b	ϵ
q_0	$\{q_0\}$	$\emptyset$	$\{q_1\}$
q_1	$\{q_2\}$	$\{q_1\}$	$\emptyset$
$q_2 \longrightarrow$	$\emptyset$	$\emptyset$	$\emptyset$

Finite automaton



ϵ-Closure



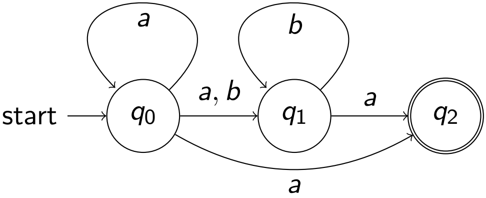
	a	b	ϵ	ϵ–Cl
$\rightarrow q_0$	$\{q_0\}$	$\emptyset$	$\{q_1\}$	$\{q_0, q_1\}$
q_1	$\{q_2\}$	$\{q_1\}$	$\emptyset$	$\{q_1\}$
$q_2^{\rightarrow}$	$\emptyset$	$\emptyset$	$\emptyset$	$\{q_2\}$

Transition into nondeterministic model

	a	b	ϵ	ϵ–Cl	a	b
$\rightarrow q_0$	$\{q_0\}$	$\emptyset$	$\{q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_1\}$
q_1	$\{q_2\}$	$\{q_1\}$	$\emptyset$	$\{q_1\}$	$\{q_2\}$	$\{q_1\}$
$q_2^{\rightarrow}$	$\emptyset$	$\emptyset$	$\emptyset$	$\{q_2\}$	$\emptyset$	$\emptyset$

Nondeterministic model

	a	b
$\rightarrow q_0$	$\{q_0, q_1, q_2\}$	$\{q_1\}$
q_1	$\{q_2\}$	$\{q_1\}$
$q_2^{\rightarrow}$	$\emptyset$	$\emptyset$



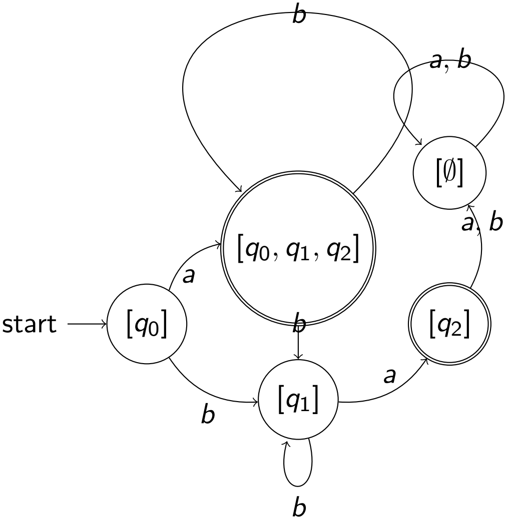
Transition into deterministic model

	a	b
$^{\rightarrow}q_0$	$\{q_0, q_1, q_2\}$	$\{q_1\}$
q_1	$\{q_2\}$	$\{q_1\}$
$q_2^{\rightarrow}$	$\emptyset$	$\emptyset$

	a	b
$[q_0]$	$[q_0, q_1 q_2]$	$[q_1]$
$[q_1]$	$[q_2]$	$[q_1]$
$[q_0, q_1, q_2]^{\rightarrow}$	$[q_0, q_1, q_2]$	$[q_1]$
$[q_2]^{\rightarrow}$	$\emptyset$	$\emptyset$

Deterministic model

	a	b
$[q_0]$	$[q_0, q_1 q_2]$	$[q_1]$
$[q_1]$	$[q_2]$	$[q_1]$
$[q_0, q_1, q_2]^{\rightarrow}$	$[q_0, q_1, q_2]$	$[q_1]$
$[q_2]^{\rightarrow}$	$[\emptyset]$	$[\emptyset]$
$[\emptyset]$	$[\emptyset]$	$[\emptyset]$



Assignments I

1. Design deterministic finite automata equivalents of the following automata:

1.1

δ	0	1	ϵ
$\rightarrow q_0$	$\{q_1\}$		$\{q_3\}$
q_1	$\{q_2\}$	$\{q_3\}$	
q_2	$\{q_3\}$		
$q_3^{\rightarrow}$	$\{q_4\}$		
q_4			$\{q_0\}$

1.2

δ	0	1	ϵ
$\rightarrow q_0$	$\{q_0, q_1\}$		$\{q_3, q_6\}$
q_1	$\{q_2\}$		
q_2		$\{q_3\}$	
q_3	$\{q_4\}$		
q_4		$\{q_5\}$	
q_5		$\{q_6\}$	
$q_6^{\rightarrow}$	$\{q_6\}$		$\{q_0\}$