Visual and cognitive aspects of transport problems

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Table of Contest

Transport (combinatorial optimization) problems

Combinatorial optimization problems

Definition

"Combinatorial optimization problems involve the search for an optimal solution among a discrete collection of possible solutions. Typically, the number of possible solutions grows exponentially with the number of elements in the problem, so “scanning all objects one by one and selecting the best one is not an option” (Schrijver, 2003, p. 1)[3]"

Combinatorial optimization problems [3]

Combinatorial optimization problem examples:

- (a) TSP instance with optimal solution
- (b) MSTP instance with optimal solution
- (c) the Fermat point (P) for a simple Steiner tree problem (shown as open circle)
- (d) GSTP instance with the interpolated open points (open circles) and the optimal solution

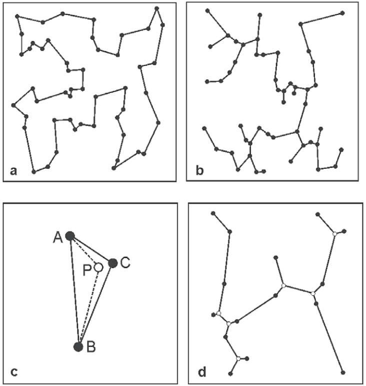

Figure 1: Examples of combinatorial optimization problems

Applications

TSP applications [3]:

- Routing of school buses to pick up children
- Home delivery of meals to elderly
- Analysis of the structure of crystals by X-ray diffraction: the necessary repositionings of the diffractometer are analogous to cities in a TSP
- Storage and picking of stock in warehousing
- Genome sequencing: software for TSP can overcome difficulties in combining partial maps of genomes

Traveling Salesman Problem (basic)

- Task: find the shortest path (tour) that passes through a set of points and returns to the origin
- Assumptions:
  - Symmetric edges
  - Given distances between each pair of cities
- Complexity: $\frac{(n-1)!}{2}$
- NP-hard problem (no polynomial-time algorithm currently exists)

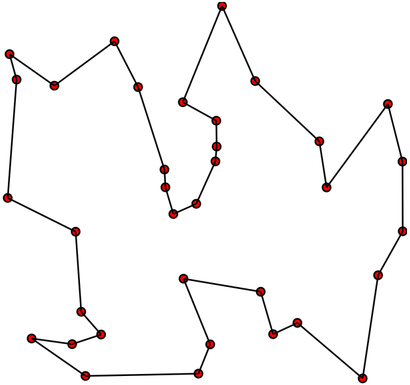

Figure 2: TSP visualisation [6]

E-TSP properties[2]

Property 1: The optimum traveling salesman tour does not intersect itself.

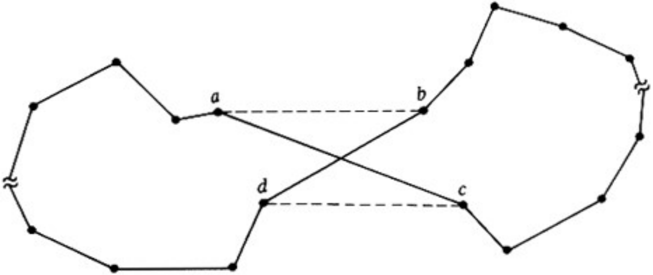

FIGURE 6.26 The traveling salesman tour can be improved by substituting the intersecting links (*a*, *c*) and (*b*, *d*) by (*a*, *b*) and (*c*, *d*).

Figure 3: E-TSP crossings rule

E-TSP properties[2]

Property 2: Let m of the n points in the Euclidean TSP define the convex hull (see below) of the points. Then the order in which these m points appear in the optimum traveling salesman tour must be the same as the order in which these same points appear on the convex hull.

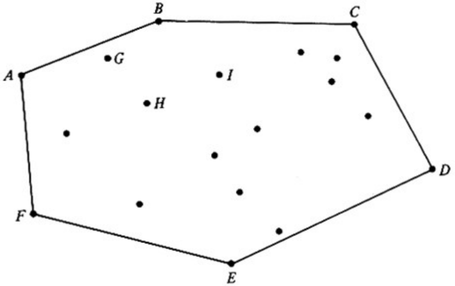

FIGURE 6.27 The convex hull of the points (*A, B, C, D, E, F, G, H, I, . . .*) is the polygon *ABCDEF*.

Figure 4: E-TSP convex hull rule

Human performance

Distinct fields of inquiry

Distinct fields of inquiry:

- Computer science - computationally less complex algorithms
- Operations research - application of human performance
- Psychology, neuroscience and cognitive science

What is measured?

Measures:

- Human performance
  - By groups
  - By age
  - By task
- Human strategies
- Correlations

Experiments

Polivanova (1974)

"Polivanova (1974) contrasted different methods of problem presentation: tables of intercity distances versus a two-dimensional plot with the cities represented as dots. The number of cities was varied from 4 to 10. On a number of measures (number of solution steps, time taken, and number of optimal solutions), participants performed better when the cities were visually represented. When the number of cities was small (four and five), the advantage of the visual presentation was reduced.[3]"

Experiments

MacGregor and Ormerod (1996)

"MacGregor and Ormerod (1996) demonstrated that in the case of 10- and 20-city problems, human subjects outperformed simple construction algorithms by an order of magnitude. Untrained human subjects, when instructed simply to draw the best route "by eye," without any real-world constraints being imposed, typically provided solutions within 1% of optimal.[3]"

Experiments

MacGregor and Ormerod (1996)

"MacGregor and Ormerod (1996) further demonstrated that (i) there was little evidence of systematic individual differences in solving either 10- or 20-city problems, and (ii) the number of cities interior to the convex hull of the problem was highly correlated with response uncertainty, a measure of problem complexity. Participants also typically connected boundary points in order and avoided crossed arcs—both features are necessary for an optimal solution to TSP. (The convex hull can be visualized as an elastic band stretched around the nodes representing cities so that each node is enclosed. Nodes in contact with the elastic band lie on the boundary.)[3]"

Convex-hull hypothesis

MacGregor and Ormerod (1996)

"The points making up a TSP can be considered as a convex set (the convex hull), and it has been shown that the optimal path for every Euclidean TSP connects adjacent points on the boundary of the convex hull in sequence, though the path may pass through interior points between adjacent boundary points (Flood, 1956). (...) If human performance followed the same principle, at least approximately, **then the complexity of a TSP should depend not so much on the total number of points, but on the number of nonboundary, or interior, points.**[4]"

Convex-hull hypothesis

Experiment 1 [4]

- Human experiment (11 X 8.5 in. sheets)
- 58 (45 finally) subjects
- 6 TSP problems with 10 points + Dantzig problem
  - Various number of interior points
- Three different orders

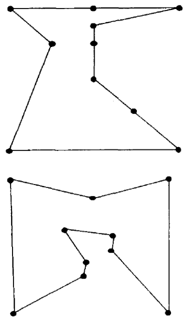

Figure 5: Two of the problems used in Experiment 1 with the points connected by the shortest paths

Convex-hull hypothesis

Experiment 1 results

Table 1
Minimum, Mean, and Maximum Path Lengths Produced by Subjects in Experiment 1, and the Corresponding *z* Values

Number of Interior Points	Subjects' Path Lengths					
	Minimum		Mean		Maximum	
	Score	*z*	Score	*z*	Score	*z*
1	530.89	4.75	535.14	4.71	582.32	4.31
2	566.22	4.51	586.11	4.32	697.14	3.27
3	559.55	4.12	574.99	3.95	647.73	3.15
4	595.30	4.32	617.94	3.99	665.00	3.31
5	558.15	4.08	575.15	3.80	638.52	2.77
6	528.51	3.49	548.83	3.11	613.33	1.93
Dantzig	758.66	3.45	787.21	3.15	870.24	2.29

Figure 6: Experiment 1 results [4]

Convex-hull hypothesis

Experiment 1 results

Table 2
Optimal Path Length and Percentage Above the Optimal for Human and Heuristic Solutions, Experiment 1

Number of Interior Points	Optimal	Percentage Above Optimal					
		Subject Minimum	NN Minimum	LA	CHCI	Subject Mean	NN Mean
1	530.89	0	0.0	0.0	0.0	0.8	0.0
2	566.22	0	0.0	0.0	0.0	3.5	0.0
3	559.55	0	0.0	5.3	2.1	2.8	6.6
4	595.30	0	0.0	11.5	2.4	3.8	10.0
5	558.15	0	4.0	6.1	1.0	3.0	9.1
6	528.51	0	5.9	2.9	0.0	3.8	10.0
Dantzig	758.66	0	0.6	3.0	2.7	3.8	10.4

Note—NN, Nearest Neighbor; LA, Largest Internal Angle; CHCI, Convex Hull, with cheapest insertion criterion.

Figure 7: Experiment 1 results [4]

Convex-hull hypothesis

Response uncertainty

Response uncertainty is defined as [4]:

$$H = \sum_{i=1}^{k} p_i(-log_2 p_i),$$

where k is the total number of connections made by subjects and p_i is probability of choosing connection i.

Convex-hull hypothesis

Response uncertainty

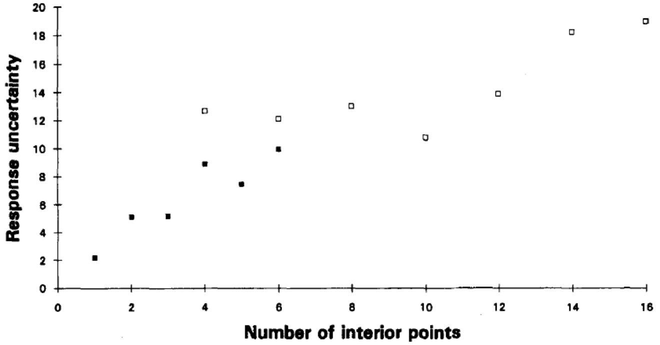

Figure 3. The response uncertainty to line connections as a function of the number of interior points. (Closed squares show the results for the 10-point problems; open squares, for the 20-point.)

Figure 8: Experiment 1 response uncertainty [4]

Convex-hull hypothesis

Number of indentations [4]

"If subjects are guided by the global properties of the convex hull, they might exhibit a preference not only for connecting points on the boundary in sequence, but also for connecting adjacent points on the boundary to each other."

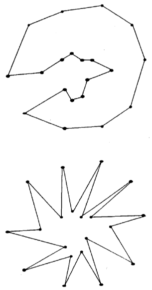

Figure 9: Two different solutions to the same 20-point problem, the one on the top with the minimum number of indents, the one on the bottom with the maximum

Convex-hull hypothesis

Number of indentations

Table 3
Expected and Observed Frequency, Relative Frequency, and Cumulative Frequency of Indentations in Problem Solutions, Experiment 1

Number of Indents	Frequencies: Expected Raw	Expected Relative	Expected Cumulative	Observed Raw	Observed Relative	Observed Cumulative
1	6	.26	.26	91	.34	.34
2	6	.26	.52	134	.50	.84
3	5	.22	.74	39	.14	.98
4	4	.17	.91	6	.02	1.00
5	2	.09	1.00	6	.00	1.00

Figure 10: Experiment 1 number of indentations [4]

Convex-hull hypothesis

Experiment 2 [4]

- Human experiment (11 X 8.5 in. sheets)
- 29 (20 finally) subjects
- 7 TSP problems with 20 points + Dantzig problem
  - Various number of interior points - 4, 6, 8, 10, 12, 14 and 16
- Two different orders

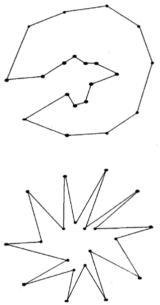

Figure 11: Two of the problems used in Experiment 1 with the points connected by the shortest paths

Convex-hull hypothesis

Experiment 2 results

Table 4
Minimum, Mean, and Maximum Path Lengths Produced by Subjects in Experiment 2, and the Corresponding *z* Values

Number of Interior Points	Subjects' Path Lengths					
	Minimum		Mean		Maximum	
	Score	*z*	Score	*z*	Score	*z*
4	707.49	8.26	724.27	8.15	816.03	7.53
6	703.89	7.16	746.32	7.16	846.50	6.50
8	737.61	7.34	762.40	7.34	830.16	6.82
10	708.93	7.60	721.81	7.60	745.03	7.39
12	692.60	7.79	719.70	7.79	828.50	6.57
14	675.80	7.68	727.36	7.68	830.68	6.42
16	599.72	6.23	644.14	6.23	734.54	5.18

Figure 12: Experiment 2 results [4]

Convex-hull hypothesis

Experiment 2 results

Table 5
Minimum Observed Path Length and Percentage Above the Minimum for Human and Heuristic Solutions, Experiment 2

Number of Interior Points	Optimal Solution	Percentage Above Optimal					
		Subject Minimum	NN Minimum	LA	CHCI	Subject Mean	NN Mean
4	703.81	3.0	4.1	3.1	2.5	5.4	8.4
6	703.89	1.2	1.2	20.1	5.3	7.3	8.8
8	725.31	1.7	3.3	13.7	5.8	5.2	9.4
10	698.83	1.4	0	6.0	4.8	3.3	9.5
12	688.33	0.6	0	23.4	3.6	4.6	7.1
14	663.61	1.9	1.5	10.0	4.4	9.6	15.4
16	593.81	1.0	15.0	0.8	0.05	8.5	21.7

Note—NN, Nearest Neighbor; LA, Largest Interior Angle; CHCI, Convex Hull, with cheapest insertion criterion.

Figure 13: Experiment 2 results [4]

Crossing-avoidance hypothesis

Rooij et al. (2003)

Rooij et al. (2003): "The purpose of this commentary is to point out the absence of evidence for the convex-hull hypothesis in the literature on human performance on E-TSP. We further suggest an alternative hypothesis, the crossing-avoidance hypothesis.[5]"

Crossing-avoidance hypothesis

Literature findings [5]

In the literature seven findings are presented as evidence for the convex-hull hypothesis (MacGregor & Ormerod, 1996; MacGregor et al., 1999, 2000):

- (1) people tend to follow the convex hull,
- (2) response uncertainty is a function of the number of interior points
- (3) people tend to produce tours without crossings,
- (4) people tend to produce tours with relatively few indentations (an indentation in a tour occurs if at least one interior point is visited between two boundary points),
- (5) performance is better when interior points are located relatively close to the convex hull than when they are located far away from the convex hull,
- (6) tours produced by a convexhull heuristic (see MacGregor et al., 2000) are close in length to tours produced by humans,
- (7) this heuristic's performance is qualitatively similar to human performance.

Crossing-avoidance hypothesis

Descriptives from previous results

Table 2
Descriptives for Tours With Crossings From Previous Research

Study	Total No. Instances	No. Tours With Crossings		No. Tours With Crossings That Follow Convex Hull		No. Tours With Nonphysical Crossings
		No.	Prop.	No.	Prop.	
MacGregor & Ormerod (1996)						
Experiment 1	315	5	0.02	0	0.00	0
Experiment 2	140	6	0.04	5	0.83	0
MacGregor et al. (1999)						
Experiment 1	103	20	0.19	6	0.30	11
Experiment 2	34	2	0.06	1	0.50	0
Experiment 3	856	65	0.08	23	0.35	4
Vickers et al. (2001)						
Experiment 1 (Group O)	108	9	0.08	3	0.33	2
Schactman (2002)						
Adult group	270	10	0.04	7	0.70	0
Total	1,826	117	0.06	45	0.38	16

Figure 14: Descriptives for Tours With Crossings From Previous Research [5]

Crossing-avoidance hypothesis

Observations [5]

- 1a. A tour that does not follow the convex hull contains at least one crossing
- 1b. A tour that does not contain any crossings follows the convex hull
- 2. There exist tours with at least one crossing that follow the convex hull
- 3. A tour that contains at least one crossing is nonoptimal (Flood, 1956).

Crossing-avoidance hypothesis

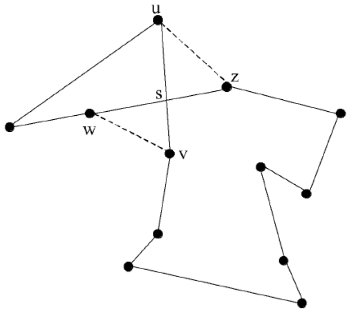

Figure 3. A tour T on some point set P with two edges (u, v) and (w, z) that cross in a point $s \notin P$. We can create a tour T' by deleting (u, v) and (w, z) from T and replacing them by (u, z) and (v, w) (dashed lines). Since $d(u, s) + d(s, z) > d(u, z)$ and $d(w, s) + d(s, v) > d(v, w)$, we know that T' is shorter than T.

Figure 15: An optimal tour follows the convex hull (Golden, Bodin, Doyle, & Stewart, 1980) [5]

Experiments

Vickers et al. (2001)

"Vickers et al. (2001, Experiment 1) used two groups of participants, one of which was given instructions to optimize tour length and the other of which was given instructions to draw the most aesthetically pleasing route through the cities. **The optimization group produced significantly shorter tours on average than the aesthetic group**. Furthermore, the best human solutions were optimal for 10- and 25-city problems, unlike a comparator elastic net algorithm, and for 40-city problems, human solutions were only approximately 2.5% above optimal, compared to 8.5% for the elastic net. An additional manipulation showed faster solution speeds for problems with more cities on the convex hull.[3]"

Experiments

Vickers et al. (2001)

"Vickers et al. (2001) found evidence for individual differences in the form of positive correlations between performances on different problems. Furthermore, in Experiment 2, they demonstrated significant correlations between various measures of solution quality and scores on Raven's Advanced Progressive Matrices. van Rooij et al. (2006) compared the performance of children aged approximately 7 and 12 with that of adults, using problems of 5, 10, and 15 nodes.[3]"

Experiments

Vickers et al. (2001)

Results for different ages:

- Adult - 42% of solutions were optimal
- 12-year-old group - 27% of solutions were optimal
- 7-year-old group - 20% of solutions were optimal

Experiments

Dry et al. (2006)

"Dry et al. (2006) used randomly generated TSPs of 10, 20, ... , 120 points and demonstrated a linear relationship between number of cities and solution time. The quality of performance showed a similar linear relationship: even for TSPs of 120 cities, human performance was only approximately 11% above optimal. The limits to human performance, in terms of the number of cities in a problem, remain to be tested empirically: it is possible, for example, that larger problems might cause catastrophic worsening of performance.[3]"

Elastic Net Algorithm

Elastic Net Algorithm

Durbin et al. (1987)

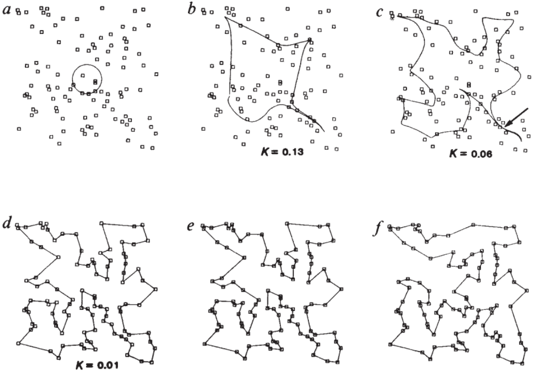

Figure 16: Example of the progress of the elastic net method for 100 cities [1]

Conclusion

Conclusion

Human performance with TSPs[3]

Humans:

- Require a visual representation of the problem
- Are good at the task, frequently outperforming simple computational algorithms in terms of finding tours close to optimal
- Demonstrate an approximately linear decrement in performance (either tour distance or speed of completion) with increasing number of cities

General conclusion

- Solution "drawn by eye" are efficient
- There are some hypotheses about human strategies
- How it can be incorporated into another transport problems?

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