04.01.24 J.D.H. Smith (Iowa State University, Ames, Iowa, U.S.A.), Quaternions and the homogeneous linearization of quasigroups.

Abstract: Linearization is a standard trick in universal algebra. An algebra (A, F) is *linear* if the underlying set A is a module over a commutative, unital ring S, and each basic operation $x_1 \dots x_n \mu$ is a linear combination $x_1 M_1 + \cdots + x_n M_n$ of the variables $x_1, \dots, x_n$ acted upon by successive S-endomorphisms of A. The identities of algebras in a variety lead to relations on the various S-endomorphisms.

A quasigroup with a pointed idempotent element is called a *pique*. If a pique is linear, its pointed idempotent element is the zero of the module, and the quasigroup operations are given respectively as the *multiplication* $x \cdot y = xR + yL$, the *right division* $(x - yL)R^{-1}$, and the *left division* $(y - xR)L^{-1}$, where R and L are automorphisms of the module corresponding to right and left multiplications by the pointed idempotent 0. While interchange of the x, y pair and the R, L pair here interchanges the left and right division, the full quasigroup *conjugacy* or *triality*, which permutes all three operations and their opposites, does not appear nicely. In particular, unlike the multiplication, the expressions for the divisions are not homogeneous in the coefficient variables R and L.

In this talk, we will present a new homogeneous method for rendering the linearization of piques naturally invariant under the action of the triality group, introducing an appropriate S-algebra generated by three invertible, non-commuting coefficient variables T_i (for $i = 1,2,3$) that is isomorphic to the group algebra of the free group on two generators. Then, we will show how this algebra has a natural quotient given by setting the square of each T_i to be -1. The quotient appears as the algebra of quaternions over the ring S, in a way which is reminiscent of how symmetric groups appear as quotients of braid groups on declaring the generators to be involutions.