Maximal Sublattices and Frattini Sublattices of Convex Geometries with $cdim = 2$

Sylvia Silberger

Hofstra University

A proper sublattice $\mathcal{M}$ of a lattice $\mathcal{L}$ is called *maximal* if it is the case that for any other proper sublattice $\mathcal{S}$, $\mathcal{M} \subseteq \mathcal{S}$ implies $\mathcal{M} = \mathcal{S}$. The *Frattini sublattice* of a lattice $\mathcal{L}$ is the intersection of all of its maximal sublattices.

In 1973, two papers, one by C. C. Chen, K. M. Koh, and S. K. Tan and the other by I. Rival both showed maximal sublattices of distributive lattices were always intervals. The former also included results descibing the Frattini sublattice of distributive lattices and more progress on that front was made by M.E. Adams, P. Dwinger, and J.Schmid in 1996. Shortly thereafter, in 1997, M. E. Adams, R Freese, J.B. Nation, and J. Schmid showed the complements of maximal sublattices of lattices in the larger class of bounded lattices were always intervals. In the same paper they also showed that the every bounded lattice is a Frattini sublattice of another bounded lattice. In this paper, Adams, Freese, Nation and Schmid posed the conjecture that spurred our research: complements of maximal sublattices of the even broader class of semi-distributive lattices are also intervals. Semi-distributive lattices are lattices that are both $\vee-$semidistributive ($SD_\vee$) and $\wedge-$semidistributed ($SD_\wedge$) and in our explorations on this matter, we arrived at the conjecture that the complements of maximal sublattices of $SD_\vee$ lattices are unions of intervals with the same minimal element and dually, the complements of maximal sublattices of $SD_\wedge$ lattices are unions of intervals with the same maximal element. As part of our work on this conjecture we focused on a sub-class of $SD_\vee$ lattices called *convex geometries*.

In this talk we describe all of the possible complements of maximal sublattices of finite convex geometries with $cdim = 2$ and use this result to give a partial description of the possible Frattini sublattices of finite convex geometries with $cdim = 2$. This is work with Kira Adaricheva, also from Hofstra University, and Anna Zamojska-Dzienio and Adam Mata, both from Warsaw University of Technology.