We look for ground state solutions to the semilinear system

$$\begin{cases} -\Delta u + \lambda_1 u = \mu_1 |u|^{p_1-2}u + \beta |v|^r |u|^{r-2}u \\ -\Delta v + \lambda_2 v = \mu_2 |v|^{p_2-2}v + \beta |u|^r |v|^{r-2}v \\ \int_{\mathbb{R}^N} u^2\,dx = \rho_1^2, \quad \int_{\mathbb{R}^N} v^2\,dx = \rho_2^2 \end{cases} \tag{1}$$

in $\mathbb{R}^N$, $N \geq 3$, where $\rho_1, \rho_2, \mu_1, \mu_2, \beta > 0$ and $2 + \frac{4}{N} < p_1, p_2, 2r < \frac{2N}{N-2}$.

Here $\lambda_1, \lambda_2 \in \mathbb{R}$ appear as Lagrange multipliers and are part of the unknown.

In order to obtain a ground state solution we minimize the energy functional over a smooth manifold obtained from a suitable linear combination of the Nehari and Pohožaev identities, which allows to get rid of the unknowns λ_1, λ_2 in this minimization process.

A major difficulty is that the subspace of radial functions of $H^1(\mathbb{R}^N)$ does not embed compactly into $L^2(\mathbb{R}^N)$, whence we do not know if the weak limit of a minimizing sequence maintains the L^2-norm. This is overcome by working in the closed L^2-ball.

The differences with the one-equation counterpart of (1), i.e.

$$\begin{cases} -\Delta u + \lambda u = \mu |u|^{p-2}u \\ \int_{\mathbb{R}^N} u^2\,dx = \rho, \end{cases}$$

will be presented. They are related to the coefficient of the coupling terms β and the dimension N.

This is joint work in progress with Jarosław Mederski.