Asymptotic Stabilization of the Flexible Beam Oscillations

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This talk is devoted to the stabilization problem for flexible beam oscillations.

The considered mechanical system consists of a simply supported flexible beam of the length l with collocated piezoelectric actuators and sensors and a controlled spring-mass attached at the point l_0 of the beam.

The beam's oscillations are modeled as the Euler–Bernoulli equation with interface and boundary conditions. Mathematical model is derived using Hamilton's principle. The equation of motion is presented in the form of an abstract differential equation with control

$$\dot{\xi} = A\xi + Bu$$

in the Hilbert space $X = \overset{\circ}{H}{}^2(0,l) \times L^2(0,l) \times \mathbb{R}^2$ with fourth-order differential operator $A : D(A) \to X$. It is proved that the operator A is the infinitesimal generator of a C_0-semigroup in X.

A feedback control is obtained in such a way that the total energy is non-increasing on the closed–loop system trajectories. It is shown, based on Lyapunov's theorem, that the obtained feedback stabilizes the coupled distributed and lumped parameter system.

The resolvent of the infinitesimal generator is obtained and is proved to be compact. So, the trajectories of the closed–loop system form a precompact set in X.

The main result is the proof that the infinite-dimensional closed–loop system has asymptotically stable trivial equilibrium under some natural assumptions on the mechanical structure. This result is obtained with the help of LaSalle's invariance principle. The spectral problem is studied for estimating the distribution of the beam's eigenfrequencies.

The observation problem for linear system is investigated for finite-dimensional projections of the original dynamical system. A Luenberger-type observer is designed for the system derived by Galerkin's approximations. The observer gain parameters are defined in such a way that the observer properly estimates the complete state vector, i.e. the error dynamics has asymptotically stable trivial equilibrium.

Results on the eigenfrequencies distribution and observer convergence are illustrated by numeric simulations.