A double phase problem involving the 1-Laplacian

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Let $\Omega \subset \mathbb{R}^n$ be a bounded Lipschitz domain and $f \in L^n(\Omega)$. We consider the following limiting double phase problem

$$\begin{cases} -\operatorname{div}(\dfrac{\nabla u}{|\nabla u|}) - \operatorname{div}(a(x)|\nabla u|^{q-2}\nabla u) &= f \text{ in } \Omega \\ u &= 0 \text{ on } \partial\Omega\,, \end{cases} \tag{1}$$

where a is a bounded function with $a(x) \geq 0$ a.e. in Ω.

As is a common theme when dealing with problems that involve the 1-Laplacian, the solution to (1) will be found as the limit of the solutions (u_p) of intermediate double phase Dirichlet problems whose lowest exponent p goes to 1. The difficulty of giving some sense to $\frac{\nabla u}{|\nabla u|}$ is overcomed by using the ideas of Anzellotti, and thus a notion of a solution should be introduced. Due to the coexistence of the 1 and the weighted q-Laplacian in the above problem, it is natural to expect that its solution should lie simultaneously in $BV(\Omega)$ and in some suitable weighted Sobolev space.

Our aim in this talk is to discuss an existence and uniqueness result of a solution to this problem, obtained under certain conditions imposed on the weight function a and the datum.

The talk is based on joint work with Nikos Yannakakis.