Numerical Solutions of Fractional Stefan Problems

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The Stefan problem, involving the tracking of a phase change interface, provides the frame work for studying moving boundary problems in a variety of physical situations. The solution of the classical one-dimensional Stefan problem predicts that in time t the phase interface goes as $s(t) \sim t^{\frac{1}{2}}$. In the presence of heterogeneity, however, anomalous behavior can be observed where the time exponent $n \neq \frac{1}{2}$. In such a case, it may be appropriate to write down the governing equations of the Stefan problem in terms of fractional order time $(1 \geq \beta > 0)$ and space $(1 \geq \alpha > 0)$ derivatives.

In this talk we start with providing a physical justification for using fractional Stefan problems and introduce a limit case problem related to the horizontal moisture movement in a porous media. Two model forms are presented; one assuming a sharp interface between the advancing wet and dry domains, the other assuming a diffuse interface, of finite thickness, across which a liquid fraction changes smoothly from a value of one to a value of zero. Analytical and numerical solutions are provided for both forms and we illustrate and discuss two important features. The first relating to how to choose the direction over which non-locality operates, i.e., what is the difference between taking non-local contribution up-stream as opposed to down-stream of the interface. The second based on the observation that, in the fractional time case $(\beta < 1)$, a solution of the fractional diffuse interface model in the sharp interface limit does not coincide with the solution of the sharp interface counterpart; negating a well know result of integer derivative Stefan problems.