We look for nontrivial solutions to the semilinear problem

$$\nabla \times \nabla \times u = f(x, u) \text{ in } \mathbb{R}^3,$$

where $u : \mathbb{R}^3 \to \mathbb{R}^3$ and $f = \nabla_u F$. We give sufficient conditions on the nonlinearity which provide a ground state solution (i.e. a nontrivial solution with the minimal energy among all the nontrivial solutions) and infinitely many geometrically distinct solutions.

The growth and asymptotic behaviour of the nonlinearity are described by an N-function which allows us to consider other model problems than the classical power type or double-power type.

After building the proper function space where to look for solutions and showing its main characteristics, we develop an abstract critical point theory, providing results that we use to solve our equation, but that may be applied to other problems.

The main difficulties are due to working in an unbounded domain and the infinite dimension of the kernel of $u \mapsto \nabla \times u$, i.e. the space of gradient vector fields. We overcome the former using a concentration-compactness argument.

At the end, we show how to solve Schrödinger's Equation using the abstract critical point theory.

This talk is based on a joint paper with Jarosław Mederski and Andrzej Szulkin.