

During the 47th International Symposium on Functional Equations in 2009, Jacek Wesółowski asked whether the identity on $[0, 1]$ is the only non-decreasing and continuous solution $\varphi: [0, 1] \rightarrow [0, 1]$ of the equation

$$(1) \quad \varphi(x) = \varphi\left(\frac{x}{2}\right) + \varphi\left(\frac{x+1}{2}\right) - \varphi\left(\frac{1}{2}\right)$$

satisfying

$$(2) \quad \varphi(0) = 0 \quad \text{and} \quad \varphi(1) = 1.$$

This question is equivalent to the following problem posed by Janusz Matkowski in [2]: Does equation (1) have a nonlinear monotonic and continuous solution $\varphi: [0, 1] \rightarrow \mathbb{R}$?

The answer to Jacek Wesółowski's question was obtained in [1]. Then class \mathcal{C} , consisting of all non-decreasing and continuous solution $\varphi: [0, 1] \rightarrow [0, 1]$ of equation (1) satisfying (2), was examined in [3, 4, 5, 6]. The purpose of this talk is to present results concerning class \mathcal{C} .

REFERENCES

- [1] T.Kania, A.Máthé, J.Morawiec, M.Rmoutil, T.Zürcher, *A functional equation*, Manuscript.
- [2] J.Matkowski, *Remark on BV-solutions of a functional equation connected with invariant measures*, Aequationes Math. 29 (1985), 210-213.
- [3] J.Morawiec, T.Zürcher, *On a problem of Janusz Matkowski and Jacek Wesółowski*, Aequationes Math. 92 (2018), 601-615.
- [4] J.Morawiec, T.Zürcher, *On a problem of Janusz Matkowski and Jacek Wesółowski, II*, Aequationes Math. 93 (2019), 91-108.
- [5] J.Morawiec, T.Zürcher, *A new approach with new solutions to the Matkowski and Wesółowski problem*, Aequationes Math. (2021), <https://doi.org/10.1007/s00010-021-00788-9>.
- [6] J.Morawiec, T.Zürcher, *The final solution to the Janusz Matkowski and Jacek Wesółowski problem*, Manuscript.