DISCRETE MATHEMATICS 1
EXERCISES
PART 2. COMBINATORIAL IDENTITIES. PARTITIONS.

1. Expand: a) $(1+x)^6$, b) $(1-x)^6$.

2. Evaluate a) $\binom{13}{6}$, b) $\binom{17}{8}$.

3. Prove that: $\binom{n+1}{3} + \binom{n-1}{3} = (n-1)^2$.

4. Show that:
   a) $\sum_{k=0}^{n} \binom{n}{k}\binom{n-k}{m-k} = 2^m\binom{n}{m}$,
   b) $\sum_{k=1}^{n} k^2\binom{n}{k} = n(n+1)2^{n-2}$.

5. By setting x equal to the appropriate values in the binomial expansion (or one of its derivates, etc.) evaluate:
   a) $\sum_{k=0}^{n} 2^k\binom{n}{k}$,
   b) $\sum k = 1^n k3^k\binom{n}{k}$.

6. Show by combinatorial arguments that:
   a) $m\binom{m+n-1}{k-1} = \sum_{i=1}^{k} i\binom{m}{i}\binom{n}{k-i}$,
   b) $\binom{2n}{2} = 2\binom{n}{2} + n^2$,
   c) $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$,
   d) $\sum_{i=0}^{n} \binom{n}{i} i = n2^{n-1}$,
   e) $\sum_{j=0}^{n} \binom{n}{j}\binom{n-j}{k-j} = \binom{n}{k}2^k$.

7. Evaluate
   a) $S(5,3)$,
   b) $S(7,5)$,
   c) $S(n,n-1)$,
   d) $S(n,2)$, where $n \geq 2$.

8. List all partitions of a 5-element set.

9. List all partitions of a 7-element set into 2 blocks.

10. Evaluate
    a)B_5,
    b) B_7.

11. Evaluate
    a) $P(9,5)$,
    b) $P(11,4)$,
    c) $P(13,8)$,
    d) $P(n,2)$, where $n \geq 2$.

12. Show that the number of partitions of an integer $n > 0$ into pairwise disjoint components is equal to the number of partitions of n into odd components.

13. Show that $P(n,3)$ is equal to the number of partitions of $2n$ into 3 components such that each of them is smaller than n.

14. Show that $P(2n,n)$ is equal to the number of all partitions of n.