

DISCRETE MATHEMATICS
EXERCISES
PART 5. CODING THEORY.

1. How many errors will detect and how many will correct the code with the following set of all code words:
a) (00000), (01011), (10101), (11110), b) (000000), (010101), (101010), (111111).
2. How many errors will detect and how many will correct the code which repeats each digit seven times?
3. Consider the linear code generated by the following matrix. Find the set of all code words. How many errors will it detect and how many will it correct? Find parity check matrix for this code. Is this code perfect?

$$a) \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$b) \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$c) \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$d) \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

4. Consider the linear code generated by the following matrix. Find the set of all code words. Find the parity check matrix for this code. Is this code perfect?

$$a) \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$b) \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

What was the sent word if the received word is (0111110).

5. Find the generator matrix for the Hamming code defined by the following parity check matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

6. Let (000000), (011101), (100111), (111010) be all code words of a linear code C .
 - a) How many generator matrices are there for C ?
 - b) Find one of them.
 - c) How many errors will this code detect and how many will it correct ?
 - d) Is the code C perfect ?
 - e) Find the parity check matrix for C .
 - f) What was sent if (111110) is received and at most one error has arisen during the transmission ?