DISCRETE MATHEMATICS
EXERCISES
PART 6. GRAPH THEORY. BASIC DEFINITIONS

1. Find all nonisomorphic grapha on 4 vertices.
2. Check if there exists a graph with the following degree sequences:
   a) $(6,2,2,2,1,1)$, b) $(5,3,3,3,3,1)$, c) $(5,4,4,3,3,2)$
   d) $(5,5,5,5,3,3)$, e) $(5,5,4,3,3,2)$, f) $(5,5,3,3,2,2)$, g) $(7,6,5,4,3,3,2)$.
3. Find a pair of nonisomorphic graphs with the same degree sequence.
4. Show that in any group of two or more people, there are always two with the same number of friends inside the group.
5. Show that in any graph G, $\delta(G) \leq \frac{2e(G)}{|G|} \leq \Delta(G)$.
6. Show that in any graph G, $e(G) \leq \binom{|G|}{2}$.
7. Show that if $e(G) > \binom{|G|-1}{2}$,then G is connected.
8. Show that if $\delta(G) \geq 2$ then G contains a cycle.
9. Show that every graph with n vertices and at least n edges contains a cycle.
10. Show that for any graph G, G is connected or $\overline{G}$ is connected.