Name ....
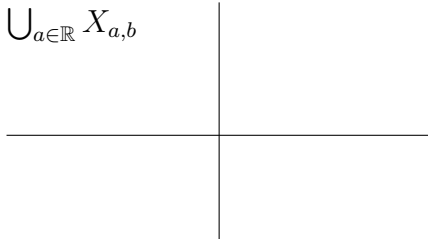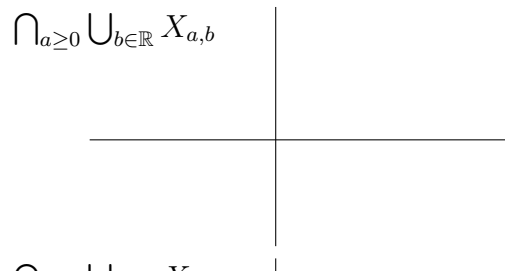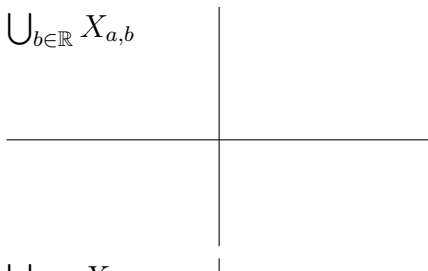
1. Write the mathematical formulas corresponding to the following statements with the help of the following signs only: propositional connectives, quantifiers, variables varied through set a) $\mathbb{N}$ b) $\mathbb{R}$ and symbols indicated in brackets

a) smallest common multiple of two odd numbers is odd $(\cdot, +, 1, =)$

b) every bounded from above quadratic polynomial has a maximum $(\cdot, +, 0, =, \geq)$

2. For $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y > a(x - b) + b\}$ where $a, b \in \mathbb{R}$. Find:

$\bigcup_{b \in \mathbb{R}} X_{a,b}$

$\bigcap_{a \geq 0} \bigcup_{b \in \mathbb{R}} X_{a,b}$

$\bigcup_{a \in \mathbb{R}} X_{a,b}$

$\bigcap_{b \in \mathbb{R}} \bigcup_{a \in \mathbb{R}} X_{a,b}$
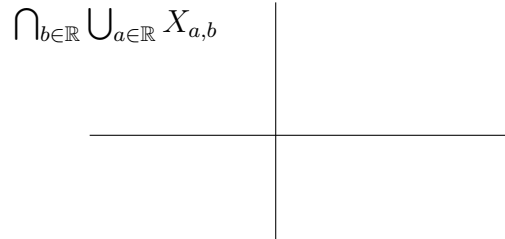
3. Is the following formula a tautology?

Transform it into CNF form (e.i. $(x_1 \vee x_2 \vee x_3) \wedge (..)... \wedge (...)$ where x_i are variable or their negations)

$[(p \Rightarrow q) \Rightarrow r] \Rightarrow (p \Rightarrow r)$

4. Are the following equalities true. Prove the true one, find a counterexample for the false one.

a) $A \cup (C \div B) = (A \cup B \cup C) \setminus [(B \cap C) \setminus A]$

b) $A \cup (C \div B) = (B \setminus C) \cup (A \setminus B) \cup (C \setminus B)$

5. Let $X = \{(x, y) : x, y \in \mathbb{R}_+\}$. $(a, b) \sim (c, d) \Leftrightarrow a - b = c - d$. Prove that $\sim$ is equivalence relation. Find equivalence classes.