Name .... CA.... row .... col....

1.	2.	3.	4.	$\sum$

1. Write the mathematical formulas corresponding to the following statements with the help of the following signs only: propositional connectives, quantifiers, variables varying through set a) set $\mathbb{N}$ b) $\mathbb{R}$ and symbols indicated in brackets

a) *there is no square root of a prime number, which is natural*$(\cdot, +, =, 1)$

b) *every positive number has exactly two square roots* $(\cdot, +, =, 1, >, 0)$

2. Prove or disprove $(x, y, z \in \mathbb{R})$

$\exists z \forall x \forall y \; z \cdot y \neq x$

3. Proof by induction

$14 | 3^{4n+2} + 5^{2n+1}$

4. For how many assignments the formula is true? Transform it into DNF form (e.i. $(x_1 \wedge x_2 \wedge x_3) \vee (..) ... \vee (...)$ where x_i are variable or their negations)

$$\sim [(q \vee p) \Rightarrow (r \vee p)] \vee \sim (p \Rightarrow q)$$

Name ....   CA.... row .... col....

1.	2.	3a.	3b	4.	$\sum$

1. For $X_{a,b} = \{(x,y) \in \mathbb{R}^2 : y > a(x-b) + \ln b\}$ where $a, b \in \mathbb{R}$. Find:

$\bigcap_{a>0} X_{a,b}$

$\bigcup_{b>0} \bigcap_{a>0} X_{a,b}$

$\bigcup_{a<0} X_{a,b}$

$\bigcap_{b>0} \bigcup_{a<0} X_{a,b}$

2. For what X the following equality holds.

$\emptyset \cup \{\emptyset, \{\emptyset, \{X\}\}\} \cup \{X, \emptyset\} \cup \{\{\emptyset\}\} \cup \{\{\emptyset\}, X\} = \{\emptyset, \{\emptyset, \{\emptyset\}\}, \{\emptyset\}\}$

3. Prove or disprove

$(A \div B) \setminus C = (A \cup B) \div C$

$(A \div B) \setminus C = [A \setminus (B \cup C)] \cup [B \setminus (A \cup C)]$

4. Let $\mathcal{A} \subseteq 2^{\mathbb{R}}$. We say that $\mathcal{A}$ has property $(\star)$ if $(\forall X, Y \in \mathcal{A})\ X \cap Y \in \mathcal{A}$. Prove or disprove:

a) family of rectangles with vertical and horizontal sides.

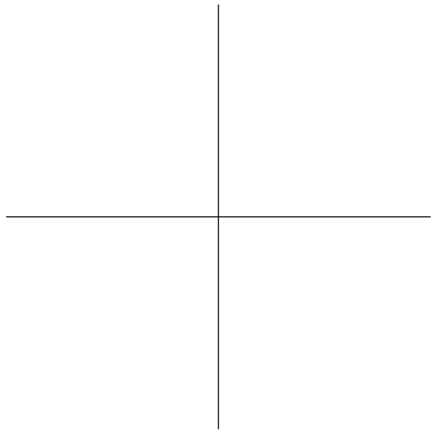
a) family of squares with vertical and horizontal sides.

Name ....

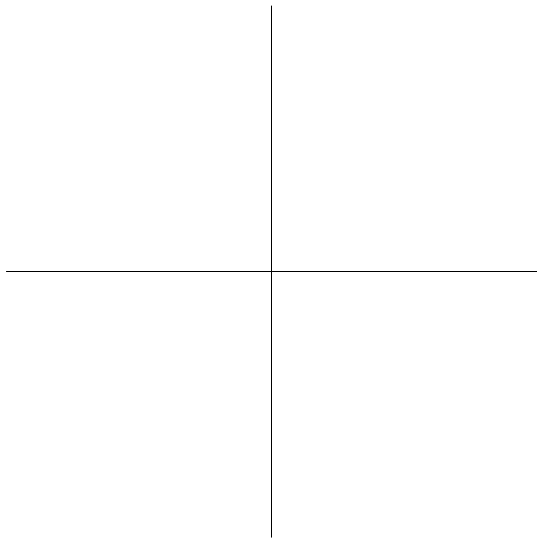
CA 1 2 row .... col....

1.	2.	3.	4.	$\sum$

1. For $(x,y),(s,t) \in \mathbb{R}^2$ let $(x,y)R(s,t) \Leftrightarrow \lfloor\sqrt{x^2+y^2}\rfloor = \lfloor\sqrt{s^2+t^2}\rfloor$. Prove R is equivalence relation. Find equivalence class $[(a,b)]_R$. Draw $[(2,2)]_R$.



2. Find $f[A]$ and $f^{-1}[f[A]]$ for $A = [-1,2] \times [0,2]$ for $f\mathbb{R}^2 \to \mathbb{R}$ where $f(x,y) = \frac{1}{x^2+y^2+1}$



3. Are given relations functions? For functions find their domain and settle if they are one-to-one functions ? $x, y, z \in \mathbb{R}$.
$(x, y)Rz \Leftrightarrow zxy = \sqrt{x^2 + y^2}$
$(x, y)Sz \Leftrightarrow 4y^2 + x^2z^2 = 4xyz$

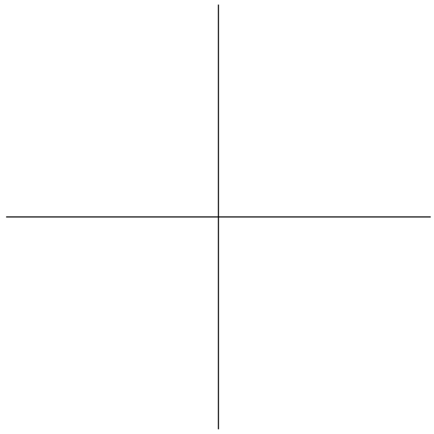
4. Is given relation $R \subset \mathbb{N} \times \mathbb{N}$ reflexive, symmetric, transitive, antisymmetric, antireflexive ? $n, k \in \mathbb{N}$
$nRk \Leftrightarrow 2|n \cdot k$.

Name ....

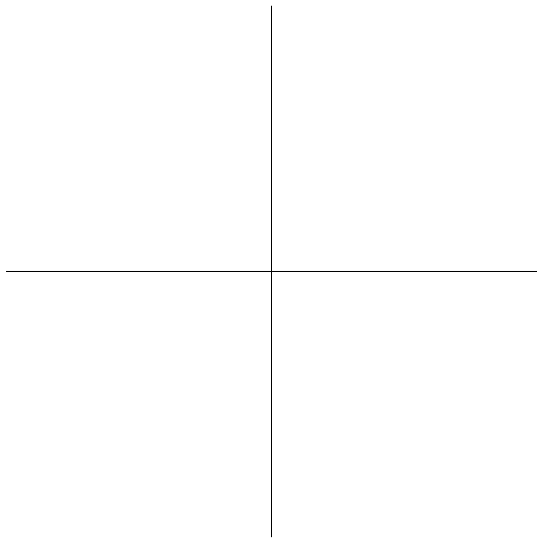
CA 1 2 row .... col....

1.	2.	3.	4.	$\sum$

1. For $(x,y),(s,t) \in \mathbb{R}^2$ let $(x,y)R(s,t) \Leftrightarrow \lfloor x+y \rfloor = \lfloor s+t \rfloor$. Prove R is equivalence relation. Find equivalence class $[(a,b)]_R$. Draw $[(1,1)]_R$.



2. Find $f[A]$ and $f^{-1}[f[A]]$ for $A = [-1,2] \times [0,2]$ for for $f\mathbb{R}^2 \to \mathbb{R}$ where $f(x,y) = \frac{1}{(x^2+1)} + y$



3. Are given relations functions? For functions find their domain and settle if they are one-to-one functions ? $x, y, z \in \mathbb{R}$.

$(x, y)Sz \Leftrightarrow y^2 + 4x^2z^2 = 4xyz$

$(x, y)Rz \Leftrightarrow zxy = x + y$

4. Is given relation $R \subset \mathbb{N} \times \mathbb{N}$ reflexive, symmetric, transitive, antisymmetric, antireflexive ? $n, k \in \mathbb{N}$

$nRk \Leftrightarrow 2 \not| n \cdot k$.