

1. Determine if the propositional expression is a tautology:

$$(p \Rightarrow (q \vee r)) \Rightarrow ((p \Rightarrow q) \vee (p \Rightarrow r))$$

2. How many numbers smaller than 1,000,000 (one million):

a) have digits in a non-decreasing order,

b) contain exactly three digits 9 and have an odd sum of digits.

3. Let us permute letters in the word: 'CALCULUS'. How many possibly obtained words do not contain a block of the same two letters?

4. *Prove*: If G is a tree then vertices of G can be numbered v_1, v_2, \dots, v_n in such a way that

$$(\forall i > 1) |\{v_k : v_k v_i \in E(G) \wedge k < i\}| = 1$$

5. Prove that in every finite (but nonempty) poset there is at least one maximal element.